

Dynamic Scoring and Monetary Policy

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Abstract

I discuss the joint effects of government-taxes and interest-rates. A fiscal authority performs 'exogenous' and 'endogenous' changes to the income-tax rate and a monetary authority sets the nominal-interest. A wedge between rates of self-financing of tax cuts and the income-tax Laffer curve arrives from the monetary system. I find a new-regime that differs from conventional monetary-fiscal policy interactions. Dynamic scoring exercises show that in the new-regime monetary-policy markedly mitigates negative output effects caused by 'exogenous tax actions' designed to reduce public-debt, altogether inducing significant welfare gains. In contrast, where public-debt is at high levels, 'exogenous tax cuts' induce welfare losses.

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1 Introduction

Dynamic scoring policy exercises usually focus on the extent to which tax cuts are self-financing if incentive feedback effects are taken into account. One of the main arguments against dynamic scoring is that it usually ignores monetary and fiscal policy reactions: the federal government faces a long-run budget constraint, so any measure that upsets the long-run budget balance must ultimately lead to further fiscal changes; and, as Auerbach (2005) argues, there is no such thing as a 'permanent' tax cut if the tax cut induces reductions in revenue. Thus, any dynamic tax scheme should specify how revenue losses are to be offset, and whether this is to be done through monetary or fiscal policy responses.

In this paper I develop a framework for determining dynamic tax schemes while taking into account incentive feedback effects, fiscal solvency, and monetary policy feedback effects. I introduce distortionary taxation and liquidity constraints to an otherwise standard Ramsey model. I obtain a policy regime that is free from deficiencies of models with lump-sum taxation, and is consistent with the results of Barro and Redlick (2011) who show that tax changes have large substitution effects on output. This regime is new in the literature as it emerges only under distortionary taxation. In this 'new' regime, liquidity traps and sovereign defaults that may emerge in conventional models, can be completely avoided.

The main findings are as follows: First, the rate at which tax cuts are self-financing is indicative as to the evolution of economic aggregates. Most obviously, rates of self-financing in monetary economies are larger than in non-monetary economies.¹ Second, various policy scenarios calibrated for the US and the EU show that the new regime induces higher welfare gains relative to conventional regimes. In this mechanism, exogenous tax actions designed to reduce high levels of debt/GDP that were built up by past deficits seem to increase welfare even though output decreases in the short run. By contrast, exogenous tax actions designed to increase long-run output end up producing higher levels of public debt unless interest rates accommodate so as to stabilize rates of self-financing. According to my calculations, policy experiments of this type induce negligible welfare gains.

¹In non monetary economies the rate of self financing of a tax cut equals one minus the slope of the income-tax Laffer curve.

It should be noted that currently, no definitive theoretical model addresses the combined effects of taxes and interest rates. The literature examining the effects of changes in tax levels on output is relatively small and mostly empirical. Early studies include Blanchard and Perotti (2002) and Perotti (1999). Recent and influential studies include Romer and Romer (2010) and Barro and Redlick (2011). Romer and Romer (2010) take a narrative approach as used in their earlier work on monetary policy² to study the macroeconomic consequences of tax changes. Focusing on the postwar US, they find that most tax changes are motivated by one of four categories: counteracting other influences on the economy; paying for increases in government spending; addressing an inherited budget deficit; and promoting long-run growth. The last two motivations are essentially unrelated to other factors influencing output, and so policy actions taken because of them are considered by Romer and Romer as 'exogenous'. Their results indicate that exogenous tax changes have very large effects on output.³ Romer and Romer (2010) also find that tax increases to reduce inherited budget deficits do not have the large output costs associated with other exogenous tax increases. However, they are largely silent concerning whether the output effects operate through incentives and supply behavior, or through disposable income and demand stimulus. Barro and Redlick (2011) study long-term US macroeconomic data to estimate the multipliers of government purchases and tax changes. They find no statistically significant effects on GDP from exogenous movements in federal revenue. In contrast, when revenue is held constant Barro and Redlick (2011) find negative effects on GDP from increases in the marginal tax rate. Thus, they conclude that changes in taxes may influence GDP mainly through substitution effects, rather than wealth effects. Previous exercises of dynamic scoring are found in Mankiw and Weinzierl (2006), Leeper and Yang (2008), and Trabandt and Uhlig (2011). All these papers abstract from monetary feedback considerations altogether.

The rest of the present paper is organized as follows: In section 2, I describe the economic environment in a manner designed to assess the joint effects of taxes and interest rates. That is, subsequent to the literature described above, and in view of Auerbach's (2005) model spec-

²See Romer and Romer (2004).

³Romer and Romer's (2010) specification implies that an exogenous increase in tax revenues of one percent of GDP lowers real GDP by almost three percent.

ification, section 2 fills in the necessary fiscal details and the assumed reaction by the Fed. It describes an economy where liquidity constraints are imposed on all transactions and where the government has access only to a distortionary taxation technology. I derive first-order conditions for optimal decision making by the representative household and by the government. I obtain a system of equations that describe the aggregate dynamics in equilibrium, and obtain conditions for global determinacy. Most importantly, the government's first order conditions coupled with a condition for global determinacy allows us to identify 'exogenous tax actions' in the context of our model.

Sections 3 and 4 discuss the determinants of equilibrium dynamics and the analytical derivation of determinacy bound in the policy parameter space. In particular I discuss the transversality condition and a concept of the Laffer curve with conjugation to the stable manifold. When assuming distortionary taxation, any change in policy affects the steady state itself. Accordingly, and in order to ensure that the economy does not converge either to liquidity traps or to equilibria where the government must renege on some of its liabilities, I carry out the analysis from a global perspective. This approach brings about a new policy regime that has been undiscovered until now because it emerges only under distortionary taxation. In this regime, Ricardian policies interact with monetary rules that respond weakly to inflation.

Section 5 studies the dynamic effects of various policy prescriptions in two dimensions. The objective of these experiments is to disentangle the effects of 'exogenous tax actions' from the dynamic effect of 'endogenous tax actions' and to obtain welfare implications of the new regime vis-a-vis conventional regimes. For this purpose, I examine transition paths to a steady state with a lower debt/GDP target with and without tax actions that increase long-run output. This policy experiment is repeated under the three regimes. Section 6 concludes the paper. All proofs are deferred to the appendix.

2 A Model with Distortionary Taxation and Liquidity Constraints

I formulate the model in continuous time to simplify the algebra and to obtain general results analytically. To economize on notations, I abstract from stochastic transitory shocks and focus on a deterministic environment. This causes no loss of generality. The model can accommodate transitory shocks to technology, preferences, and policy instruments by representing the relevant parameters as stochastic processes.

2.1 The Households Sector

The economy is closed and populated by a continuum of identical infinitely long-lived households, with measure one. The representative household enjoys consumption and is endowed with perfect foresight and one unit of time per "period". The representative household inelastically supplies its endowment of labor, so its lifetime utility is given by

$$U_t = \int_t^{\infty} e^{-\rho s} u(c_s) ds \quad (1)$$

where $\rho > 0$ denotes the rate of time preference, c_s denotes consumption per capita, and $u(\cdot)$ is twice differentiable, strictly increasing, strictly concave, and satisfies the usual limit conditions. Production takes place in a competitive sector via a constant-returns-to-scale production technology $f(k_t)$ where k_t denotes per-capita capital which depreciates at a rate δ . Finally, $f(k_t)$ is concave and twice differentiable. Money enters the economy via a liquidity constraint on all transactions. Let m_t denote the per-capita stock of money denominated in the consumption good, and let ν denote money velocity. Then a formal representation of the liquidity constraint is

$$c_t + I_t \leq \nu m_t \quad (2)$$

where I_t denotes per-capita investment.⁴ We assume that the government has access only to distortionary taxation and that deficits are financed via bond creation. As a consequence, the representative household's budget constraint becomes

$$c_t + I_t + \dot{b}_t + \dot{m}_t = (R_t - \pi_t)b_t - \pi_t m_t + (1 - \tau_t)f(k_t) + T_t \quad (3)$$

where $\tau_t \in [0, 1]$ is the income tax rate, b_t is a real measure of the stock of non-indexed government bonds, R_t is the nominal rate of interest, π_t is the rate of inflation, and T_t is a real lump-sum transfer. Capital accumulates according to

$$\dot{k}_t = I_t - \delta k_t. \quad (4)$$

Altogether, the household maximizes its lifetime utility given by (1) subject to the constraints

(2)-(4), with a borrowing constraint such that $\lim_{t \rightarrow \infty} a_t^H e^{-\int_0^t [R_s - \pi_s] ds} \geq 0$ where $a_t^H \equiv b_t + m_t$. Each household chooses sequences of $[(c_t, I_t, m_t)]_{t=0}^{+\infty}$ so as to maximize its lifetime utility, taking as given the initial stock of capital k_0 , the initial stock of financial wealth a_0^H , and the time path $[(\tau_t, T_t, R_t, \pi_t)]_{t=0}^{+\infty}$ which is exogenous from the household's viewpoint.

The necessary conditions for an interior maximum are

$$u'(c_t) = \lambda_t \left(1 + \frac{1}{\nu} R_t\right) \quad (5a)$$

$$\mu_t = u'(c_t) \quad (5b)$$

$$\zeta_t = \frac{1}{\nu} R_t \lambda_t \quad (5c)$$

$$\zeta_t (\nu m_t - c_t - I_t) = 0; \zeta_t \geq 0 \quad (5d)$$

⁴Let $\frac{1}{\nu}$ denote the inverse of money velocity, then a requirement that $\int_t^{t+\frac{1}{\nu}} [c(s) + I(s)] ds \leq m_t$ formalizes the liquidity constraint. A Taylor series expansion gives $\int_t^{t+\frac{1}{\nu}} [c(s) + I(s)] ds = \frac{1}{\nu} [c(t) + I(t)] + \frac{1}{2} \left(\frac{1}{\nu}\right)^2 [\dot{c}(t) + \dot{I}(t)] + \dots$; and $\frac{1}{\nu} (c + I) \leq m$ can be interpreted as a first-order approximation.

where λ_t, μ_t are time-dependent co-state variables interpreted as the marginal valuations of financial wealth and capital, respectively; ζ_t is a time-dependent Lagrange multiplier associated with the liquidity constraint; and equation (5d) is a Kuhn-Tucker condition.

Restricting attention to positive nominal interest rates, equations (5c)-(5d) imply that ζ_t is positive, which in turn implies that the liquidity constraint is binding. Second, and after substituting $m_t = \frac{1}{\nu}(c_t + I_t)$ and $a_t^H = b_t + m_t$ into equation (3), the state and co-state variables must evolve according to

$$\dot{\lambda}_t = \lambda_t [\rho + \pi_t - R_t] \quad (6)$$

$$\dot{\mu}_t = -\lambda_t(1 - \tau_t)f'(k_t) + (\rho + \delta)\mu_t \quad (7)$$

$$\dot{k}_t = I_t - \delta k_t \quad (8)$$

$$\dot{a}_t^H = (R_t - \pi_t)a_t^H + (1 - \tau_t)f(k_t) + T_t - (c_t + I_t) \left(1 + \frac{1}{\nu}R_t\right) \quad (9)$$

Solving equation (9) yields that the household's intertemporal budget constraint is of the form

$$0 \leq \lim_{t \rightarrow \infty} e^{-\int_0^t [R_s - \pi_s] ds} a_t^H = a_0^H + \int_0^\infty e^{-\int_0^t [R_s - \pi_s] ds} \left[(1 - \tau_t)f(k_t) + T_t - (c_t + I_t) \left(1 + \frac{1}{\nu}R_t\right) \right] dt$$

and the condition that its decisions are dynamically efficient yields the household's transversality condition

$$\lim_{t \rightarrow \infty} a_t^H e^{-\int_0^t [R_s - \pi_s] ds} = 0. \quad (10)$$

Equations (6) – (10) fully describe the optimal program of a representative household for which the time path $[(\tau_t, T_t, R_t, \pi_t)]_{t=0}^{+\infty}$ is exogenously given.

2.2 The Government

The government consists of a fiscal authority and a monetary authority. The consolidated government prints money, issues nominal bonds, collects taxes to the amount of $\tau_t y_t$ where y_t is output, and rebates to the households a lump-sum transfer T_t . Its dollar-denominated budget constraint is therefore given by $R_t B_t + P_t T_t = \dot{M}_t + \dot{B}_t + P_t \tau_t y_t$, where P_t is the nominal price of a consumption bundle, \dot{M}_t and \dot{B}_t are net changes in the money and bond supply, respectively, and R_t is the nominal interest paid over outstanding debt. Dividing both sides of the nominal budget constraint by P_t and rearranging yields that government liabilities, denoted by $a_t^G \equiv \frac{M_t}{P_t} + \frac{B_t}{P_t}$, evolve according to

$$\dot{a}_t^G = \underbrace{(R_t - \pi_t) a_t^G}_{\text{interest payments on the debt}} - \underbrace{R_t m_t}_{\text{seigniorage}} + \underbrace{T_t - \tau_t y_t}_{\text{primary deficit}} \quad (11)$$

where $\pi_t \equiv \widehat{\frac{P_t}{P_t}}$ and the hatted time derivative is a right derivative, referring to expected inflation from now on. Equation (11) shows that since the consolidated budget is not necessarily balanced at every instant, deficits (surpluses) are financed via increments (decrements) to government debt. As a result, government liabilities increase with the primary deficit and with the real interest paid over outstanding debt, and decrease with seigniorage.

Fiscal and Monetary Policies

We assume an exogenous path for lump-sum transfers $T_t = \bar{T}$. Following Barro (1979), we model the government as wanting to minimize the deadweight losses from the distortions in the economy, modeled as proportional to $(\tau_t y_t)^2$ and $(R_t m_t)^2$, the squares of total tax revenue and seigniorage, respectively. Since we scrutinize a monetary economy, we add a concern with wide swings in inflation, leading to the objective function

$$-\frac{1}{2} \int_0^{\infty} e^{-\rho t} [(\tau_t y_t)^2 + (R_t m_t)^2 + \pi_t^2] dt \quad (12)$$

The government then maximizes (12) by choosing sequences of τ_t, R_t , and π_t subject to the flow budget constraint (11). The resulting first order conditions are

$$\tau_t y_t = -\Gamma_t \quad (13)$$

$$R_t m_t + \Gamma_t = \Gamma_t \frac{a_t^G}{m_t} \quad (14)$$

$$\pi_t = -\Gamma_t a_t^G \quad (15)$$

$$\frac{\dot{\Gamma}_t}{\Gamma_t} = \rho + \pi_t - R_t \quad (16)$$

where $\Gamma_t \equiv \Gamma(a_t^G)$ is a co-state variable associated with government debt and can be thought of as the marginal deadweight loss accrued from an additional 'unit' of government debt. Since Γ_t measures marginal losses that are increasing, in absolute value, with government debt, one can conclude that $\Gamma_t < 0$ and that $\frac{d\Gamma_t}{da_t^G} < 0$.

The first order condition with respect to the income-tax rate, eq. (13), states that the marginal tax revenue should equal the marginal loss due to an increase in government debt. Eq. (13) also implies that the income-tax rate is responsive to output and government debt: $\tau(y_t, a_t^G) = -\frac{\Gamma_t}{y_t}$. From what we know about the co-state variable of government debt, Γ_t , we conclude that $\frac{\partial \tau(y_t, a_t^G)}{\partial y_t} < 0$, $\frac{\partial \tau(y_t, a_t^G)}{\partial a_t^G} > 0$ and the simplest rule to approximate $\tau(y_t, a_t^G)$ reads⁵

$$\tau(y_t, a_t^G) = \tau^* + \beta \frac{y_t - y^*}{y_t} + \gamma \frac{a_t^G - a^*}{a^*} \quad (17)$$

There is much documented empirical relevance for fiscal rules in the literature. Romer and Romer (2010) find that most tax changes have an identifiable motivation that falls into one of four broad categories: a) offsetting a change in government spending; b) offsetting some factor other than spending likely to affect output in the near future; c) dealing with inherited budget deficit or debt; or d) achieving some long-run goal such as a higher level of output. They separate tax changes into two broad categories: 'endogenous tax actions', which are those taken in response to other factors likely to affect output in the short run, and

⁵This specification also concurs with the empirical findings of Leeper and Yang (2008) and Leeper et al. (2010).

'exogenous tax actions', which are those taken for any other reason. 'Exogenous tax actions' are those not taken to offset factors pushing output away from normal. The most common such action might be a tax cut motivated by a belief that lower marginal tax rates will raise output in the long run. Such an action is fundamentally different from the countercyclical actions because the goal is to raise the long-run output and not to offset shocks acting to reduce output temporarily. Another 'Exogenous tax action' might be a temporary tax hike motivated by a belief that a lower debt level will raise output in the long run. Observe eq. (17). Then, in the terminology of Romer and Romer (2010), β represents the 'endogenous' element of tax actions, while γ and τ^* represent 'exogenous' elements.

Note the first order condition with respect to seigniorage, eq. (14). It states that the marginal loss from financing debt via bond creation should equal the marginal loss from financing debt via money creation. Finally, note that inflation is a tax imposed on nominal assets. From the view point of the government inflation reduces the rate of growth of its liabilities. Eq. (15), states that this lump sum tax should equal the loss due to an increase in the real value of government debt. Altogether eq. (15)-(16) show how the rate of inflation responds to the fiscal and monetary instruments. Taking time derivative from eq. (15), substituting in rates of growth for government debt and its co-state variable obtained from equations (11) and (16) we obtain that $\frac{\dot{\pi}_t}{\pi_t} = \rho + \frac{T_t - \tau_t y_t - R_t m_t}{a_t^G}$ which means that inflation grows with the real rate of interest and the total deficit. This implies that given the state variables and the fiscal stance, a central bank can only increase the nominal rate of interest if it wishes to respond to undesired increases in inflation. The simplest rule to approximate $R(\pi_t)$ thus reads

$$R(\pi_t) = \rho + \pi^* + \alpha(\pi_t - \pi^*) \quad (18)$$

In Leeper's (1991) terminology a monetary rule that exhibits $\alpha > 1$ is called an 'active' monetary policy, whereas $\alpha < 1$ corresponds to a 'passive' monetary policy. While most of the literature emphasizes the active stance as a prescription for sound policy, Benhabib et al. (2001, 2002) focus on its perils. They show that once the zero bound on nominal interest rates is taken into account, 'active' rules give rise to unintended liquidity traps. They propose several fiscal and monetary policies that preserve local uniqueness of equilibrium while ruling

out the deflationary expectations that can lead an economy into a liquidity trap. These prescriptions primarily involve regimes switching once the interest rate approaches the zero lower bound.

2.3 General Equilibrium

In equilibrium, a) the goods market clears

$$f(k_t) = y_t = c_t + I_t, \quad (19)$$

b) the money market clears

$$m_t = \frac{1}{\nu} (c_t + I_t), \quad (20)$$

and c) the assets market clears $a_t \equiv a_t^G = a_t^H$.

Using the monetary policy rule and the fiscal rules, imposing market clearing conditions, and assuming that the elasticity of intertemporal substitution in consumption is constant, we arrive at the following characterization of the general equilibrium of the economy:

Proposition 1 *In equilibrium with distortionary taxation and liquidity constraints, the aggregate dynamics satisfy the following ODE system:*

$$\frac{\dot{c}_t}{c_t} = \sigma \left\{ \left[\frac{1 - \tau(f(k_t), a_t)}{1 + \frac{1}{\nu} R(\pi_t)} f'(k_t) - \delta \right] - \rho \right\} \quad (21)$$

$$\dot{\pi}_t = \frac{\nu + R(\pi_t)}{\alpha} \left\{ [R(\pi_t) - \pi_t] - \left[\frac{1 - \tau(f(k_t), a_t)}{1 + \frac{1}{\nu} R(\pi_t)} f'(k_t) - \delta \right] \right\} \quad (22)$$

$$\dot{k}_t = f(k_t) - c_t - \delta k_t \quad (23)$$

$$\dot{a}_t = [R(\pi_t) - \pi_t] a_t + T_t - \left[\tau(f(k_t), a_t) + \frac{1}{\nu} R(\pi_t) \right] f(k_t). \quad (24)$$

Equation (21) is an Euler equation, where $\sigma > 0$ denotes the elasticity of intertemporal substitution in private consumption. In our economy the marginal product of capital is distorted by the income-tax and liquidity constraints. Notice that with no distortions equation (21) becomes the familiar Ramsey-type Euler equation. Equation (22) was obtained by tak-

ing a time derivative from the first-order condition (5a) and substituting in equation (6). It corresponds to a Fisher equation in which the nominal rate of interest varies with expected inflation and the real rate of interest. It shows that since capital and bonds are perfect substitutes at the private level, in equilibrium the distorted marginal product of capital net of depreciation must equal the real interest received from holding a risk-free bond minus the expected change in inflation after the policy response to inflation is internalized. Finally, equations (23)-(24) were obtained by substituting market clearing conditions (19)-(20) into equations (8)-(9). At this point, the set of equations (21)-(24) internalizes the government's policies (17)-(18) and market clearing conditions and can therefore be viewed as the solution to the central planner's problem. At this point we can start to characterize the equilibrium,

Definition 1 *An equilibrium with distortionary taxation and liquidity constraints is a set of sequences $\{[(c_t, \pi_t, k_t, a_t, \tau_t, T_t, R_t)]_{t=0}^{+\infty}\}$ satisfying (21)-(24) given $k_0, a_0 > 0$.*

2.4 Steady State Equilibrium

It follows from equation (21) that in a steady state,

$$f'(k^*) = (\rho + \delta) \frac{1 + \frac{1}{\nu} R^*}{1 - \tau^*}, \quad (25)$$

where τ^* denotes a long-run income-tax rate and R^* is a steady-state rate of interest. We can see the distorting effect of income taxes and interest rates on long-run output as the marginal product of capital increases with both distortions. From equations (22) and (25), R^* must satisfy

$$R^* = \rho + \pi^* \quad (26)$$

where π^* is the long-run rate of inflation. Equation (23) implies that the steady-state consumption is

$$c^* = f(k^*) - \delta k^*. \quad (27)$$

Finally, equation (24) shows that in a steady-state equilibrium, government liabilities must satisfy $a^* = \frac{1}{\rho} [f(k^*)(\tau^* + \frac{1}{\nu} R^*) - \bar{T}]$. Let $\tilde{a}^* \equiv \frac{a^*}{f(k^*)}$, $\tilde{T}^* \equiv \frac{\bar{T}}{f(k^*)}$ denote debt/GDP and

transfers/GDP in the steady state, respectively. Then, we obtain that a sustainable debt must satisfy

$$\tilde{a}^* = \frac{1}{\rho} \left[\tau^* + \frac{1}{\nu} R^* - \tilde{T}^* \right]. \quad (28)$$

Note that an equilibrium trajectory $[(c_t, \pi_t, k_t, a_t, \tau_t, T_t, R_t)]_{t=0}^{+\infty}$ should converge to the steady state $(c^*, \pi^*, k^*, a^*, \tau^*, T^*, R^*)$, whereas the system of equations (21) - (24) determines only four dimensions of the steady state. It is thus apparent at the outset that our model may exhibit global indeterminacy.

Definition 2 (Global Determinacy) *The equilibrium displays global determinacy if the system (21) - (24) has a unique stationary solution $(c^*, \pi^*, k^*, a^*, \tau^*, T^*, R^*)$.*

This leads us to the following proposition:

Proposition 2 *A necessary condition for global determinacy is that the government proclaims three 'exogenous' targets.*

Proposition 2 shows that the steady state is sustained only if the revenues from taxes and seigniorage equal the sum of transfers and debt service. Thus, as equation (28) links $\pi^*, \tau^*, a^*, \bar{T}$ to a balanced budget condition, three targets should be specified 'exogenously', and the fourth is implied by the stipulation to run a balanced budget in the steady state. Specifically,

Corollary 1 *Where the government proclaims the 'exogenous target' $(\pi^*, \tau^*, \tilde{a}^*)$, the long-run surplus/GDP is implied according to $\tau^* - \tilde{T}^* = \rho \tilde{a}^* - \frac{1}{\nu} R^*$.*

Corollary 1 follows directly from (28). In the rest of the paper we assume that the government chooses $(\pi^*, \tau^*, \tilde{a}^*)$ as 'exogenous targets'. Then, from Corollary 1 we get that the 'exogenous targets' imply the long-run level of transfers/GDP, \tilde{T}^* , and as a result the implied long-run level of lump-sum transfers is $\bar{T} = \tilde{T}^* f(k^*)$.

3 Determinants of Equilibrium Dynamics

In this section we pursue policy regimes that bring about a unique bounded rational expectations equilibrium.

3.1 The Transversality Condition

The system (21)-(24) shows that in equilibrium, markets clear and households rationally internalize the policy rules. However, to characterize equilibrium correctly we must impose the condition that the household's intertemporal budget constraint holds with equality. Note, however, that the choice of γ determines the rate of growth of government debt. In order to study the effect of γ on the evolution of government debt, we substitute the fiscal rule (17) into (24) and obtain that government debt evolves according to

$$\dot{a}_t = \left[R(\pi_t) - \pi_t - \gamma \frac{f(k_t)}{a^*} \right] a_t - f(k_t) \left[\tau^* + \beta \frac{f(k_t) - f(k^*)}{f(k^*)} - \gamma + \frac{1}{\nu} R(\pi_t) \right] + \bar{T}. \quad (29)$$

Solving equation (29) for a_t and letting $t \rightarrow \infty$, we arrive at

$$\lim_{t \rightarrow \infty} Q_t a_t = a_0 - \int_0^{\infty} Q_s X_s ds \quad (30)$$

where $Q_t \equiv e^{-\int_0^t [(R(\pi_s) - \pi_s) - \gamma \frac{f(k_s)}{a^*}] ds}$, $X_s \equiv \left\{ \left[\tau^* + \beta \frac{f(k_s) - f(k^*)}{f(k^*)} - \gamma + \frac{1}{\nu} R(\pi_s) \right] f(k_s) - \bar{T} \right\}$.

X_s is the surplus at instant s and Q_s is its respective discount factor. As we assume that transfers are constant, the surplus flow has two components. The first comprises revenues from taxing all sources of income in the economy. Note that the tax rate includes an automatic stabilizer component, which we modeled as a response of the income-tax rate to deviations of output from the steady state. The second surplus flow component comprises seigniorage revenues resulting from the liquidity constraint on all transactions in the economy. Also note that the discount factor has two components. The first stems from monetary

policy and equals the real rate of interest, while the second stems from fiscal policy and attaches a growth premium to the surplus flow. Equation (30) has several implications. As we know, the left-hand side of the equation must equal zero in equilibrium. If γ is large enough, real debt will shrink back to its long-run level and the transversality condition is ensured. By contrast, a too-small γ may produce the impression that the government lets its debt grow too fast. Episodes where the government is growing at a faster rate than tax revenues may raise the possibility of a sovereign default. In what follows I reestablish some of the previous results in the context of our model and highlight new results that emerge from our model. I start by defining Ricardian and non-Ricardian policy regimes.

Definition 3 *Fiscal policy is considered Ricardian if the exogenous sequences and feedback rules that specify the policy regime imply that the transversality condition necessarily holds for any initial level of government debt. Fiscal policy is considered non-Ricardian if for any sequence $[(c_t, \pi_t, k_t, a_t, \tau_t, T_t, R_t)]_{t=0}^{+\infty}$ satisfying (21)-(24) there exists a unique valuation for government debt $a(0) > 0$ that is consistent with equilibrium.*

Equation (30) has the following implications:

Proposition 3 *Fiscal rules are Ricardian if and only if $\gamma \geq \rho\tilde{a}^*$.*

Where $\gamma \geq \rho\tilde{a}^*$, both sides of (30) may grow to infinity since Q_t is not contracting. Note, however, that eq. (30) reads

$$\lim_{t \rightarrow \infty} e^{-\int_0^t [R(\pi_s) - \pi_s] ds} a_t = \lim_{t \rightarrow \infty} \left[\frac{a_0 - \int_0^t Q_s X_s ds}{e^{\int_0^t \gamma \frac{f(k_s)}{a^*} ds}} \right]. \quad (31)$$

In this formulation one can see that $\gamma \geq \rho\tilde{a}^*$ implies that both the numerator and the denominator of the right-hand side of eq. (31) expand to infinity, and it is important to determine which will grow faster. It is straightforward to show via L'Hospital's law that where $\gamma \geq \rho\tilde{a}^*$, the limit of the expression on the right-hand side of eq. (31) is zero. Thus, policy rules that exhibit $\gamma \geq \rho\tilde{a}^*$ ensure that the household's transversality condition is not

violated. Policy stances of this type imply that the government ensures that its liabilities will converge back to the target. In particular, in this regime the government is committed to raising taxes as much as needed so as to ensure fiscal solvency for any given initial level of government debt.

Proposition 4 *Fiscal rules are non-Ricardian if and only if $\gamma < \rho\tilde{a}^*$. In this case the initial real value of government debt, a_0 , must jump so as to satisfy $a_0 = \int_0^\infty Q_s X_s ds$.*

Where $\gamma < \rho\tilde{a}^*$, Q_t is contracting and the value $\int_0^\infty Q_s X_s ds$ is not necessarily equal to a_0 . In such cases, the right-hand side of eq. (30) is non-zero, which implies that the transversality condition does not hold. In this case solvency is brought about only via changes to a_0 so as to equate the right-hand side of eq. (30) to zero. The novelty of Proposition 4 is twofold. First, fiscal policy should be considered *non-Ricardian* where the response of the income-tax rate to government debt is less than $\rho\tilde{a}^*$. This result has an important implication for the choice of policy targets. The debt-to-GDP target now influences the threshold under which fiscal policy is considered non-Ricardian. Second, real determinacy becomes a necessary condition for pinpointing the value of government debt where fiscal policy is non-Ricardian. Where taxes are lump sum and future surpluses are independent of real allocation, it is possible to pinpoint the initial value of government debt based on the surpluses alone, and whether there is a unique trajectory or multiplicity of trajectories of real allocations has no relevance to our ability to calculate the present value of future surpluses. In contrast, where the government has access only to distortionary taxation, tax revenues become a feature of equilibrium. Moreover, in our model output and the tax rate are determined simultaneously in equilibrium. Hence, future surpluses depend on the entire equilibrium trajectory.

3.2 The Laffer Curve and the Stable Manifold

In what follows I propose terminology required to discuss dynamic Laffer curves in the context of monetary and fiscal policy interactions. Let $\dot{x}_t = g(x_t)$ denote the system of

equations (21)-(24). Then a linear approximation near the steady state reads

$$\dot{x}_t = B \times (x_t - x^*) \quad (32)$$

and we obtain analytically that the product of system's eigenvalues equals⁶

$$- [\tilde{c}^* \nu \sigma \rho f_k^{*2}] \frac{\alpha-1}{\alpha} \left[\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho \tilde{a}^*} \left(\frac{1}{\nu} R^* - \frac{\tau^* \varphi^*}{1-\varphi^*} \right) \right]$$

where $\varphi_{(\tau_t, y_t)} \equiv \frac{\partial \ln(\tau_t y_t)}{\partial \ln(\tau_t)} = 1 + \frac{\partial \ln(y_t)}{\partial \ln(\tau_t)}$ denotes the marginal revenue generated from an increase in taxes, and one can interpret $\varphi_{(\tau_t, y_t)}$ as the slope of the Laffer curve. The second term is negative as higher taxes decrease output, so the elasticity of tax revenue with respect to tax rates is less than one. In this economy $y_t = f(k_t)$, accordingly $\varphi_{(\tau_t, y_t)} = 1 + \frac{\tau_t}{f(k_t)} \frac{\partial f(k_t)}{\partial \tau_t} = 1 + \frac{\tau_t}{f(k_t)} f'(k_t) \frac{dk_t}{d\tau_t}$. It is straightforward to obtain⁷ that $\frac{dk^*}{d\tau^*} = \frac{1}{1-\tau^*} \frac{f'(k^*)}{f''(k^*)}$, and therefore the slope of the Laffer curve near the steady state reads

$$\varphi_{(\tau^*)} = 1 + \frac{\tau^*}{1-\tau^*} \frac{[f'(k^*)]^2}{f(k^*)f''(k^*)} \quad (33)$$

The slope is related to the degree to which a tax cut is self-financing, defined as the ratio of additional tax revenues due to general equilibrium effects and the lost tax revenues due to the tax cut. More formally, adopting the terminology of Trabandt and Uhlig (2011) with adjustments to a monetary economy, the degree to which a tax cut is self-financing, denoted by \mathcal{RSF} , is calculated as

$$\mathcal{RSF} = 1 - \frac{1}{f(k^*)} \frac{d[f(k^*)(\tau^* + \frac{1}{\nu} R^*)]}{d\tau^*}$$

where $f(k^*)(\tau^* + \frac{1}{\nu} R^*)$ are total tax revenues in the steady state. If there were no endogenous changes in allocations following a tax change, the loss in tax revenue due to a one-percentage-point reduction in the tax rate would be one percent of $f(k^*)$, and the self-financing rate would calculate to 0. In a non-monetary economy, at the peak of the Laffer curve, tax revenue would not change at all in the wake of a one-percentage-point reduction in the tax rate, and the self-financing rate would be 1. This self-financing rate would become larger than 1 beyond the peak of the Laffer curve. Note, however, that in our economy seigniorage is a source of revenue. Thus, tax cuts may affect seigniorage revenues via general

⁶See the preliminary section in the technical appendix.

⁷By applying the implicit function theorem on equation (25).

equilibrium effects. All in all, we find that the rate of self-financing near the steady state depends on the elasticity of tax revenues, the tax-rate target, and the inflation target, and reads

$$\mathcal{RSF}^* = 1 - \frac{\varphi^* - 1}{\tau^*} \left[\frac{1}{\nu} R^* - \frac{\tau^* \varphi^*}{1 - \varphi^*} \right]. \quad (34)$$

Note that in a non-monetary economy the rate of self-financing should equal one minus the elasticity of tax revenues. As a result, the measure of $\mathcal{RSF}^* - 1$, in non-monetary economies, switches signs at the peak of the Laffer curve.⁸ Note, however, that this is not the case in our model economy. As eq. (34) indicates, $\mathcal{RSF}^* - 1$ will switch signs at the peak of the Laffer curve only if the steady-state nominal interest rate is zero. Otherwise, the sign switches where the elasticity of tax revenues is still positive and equals $\frac{1}{\nu} R^*$. We can thus conclude that economies where the long-run nominal rate of interest is positive exhibit greater rates of self-financing than economies with a zero nominal interest rate.

At this point it is helpful to introduce a notation that will be used in the rest of the paper:

$$\phi_\alpha \equiv \frac{\alpha - 1}{\alpha}$$

$$\phi_\beta \equiv \beta + \frac{\tau^*}{1 - \varphi^*}$$

$$\phi_\gamma \equiv \frac{\gamma}{\rho \bar{a}^*}$$

ϕ_α describes 'endogenous' interest responses to deviations of inflation from its 'exogenous' target, ϕ_β describes an 'endogenous' tax response to deviations of output from its 'exogenous' target, and ϕ_γ describes an 'exogenous' tax response to excess debt accrued by past deficits. One can think of ϕ_α as the net effect of the monetary response on the real rate of interest when inflation is above target. Thus, a negative ϕ_α implies that the monetary policy lets the real rate of interest drop below its long-run level when inflation is above its 'exogenous' target. $\phi_\alpha > 0$ corresponds to Taylor rules, and monetary rules that exhibit $\phi_\alpha < 0$ are considered 'passive'. Similarly, one can think of ϕ_γ as the effect of secondary deficits on tax hikes. According to this interpretation, $\phi_\gamma = 1$ should imply that only the interest paid on the government debt is financed via tax hikes, $\phi_\gamma > 1$ should imply that when debt increases above its long-run level income taxes rise more than needed to fully offset all the excess interest payments, and according to Proposition 3 such fiscal rules are Ricardian.

⁸See for example Uhlig and Trabandt (2011).

Equivalently, $\phi_\gamma < 1$ implies that when debt increases above its long-run level, income taxes may rise. However, in this case tax revenues are not enough to stop the debt from growing. According to Proposition 4, such rules are non-Ricardian.

Let r_i $i = 1, \dots, 4$ denote the eigenvalues of B . Then having the expressions for the trace and determinant of B , we obtain that

$$r_1 r_2 r_3 r_4 = -\kappa \phi_\alpha \left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1 - \varphi^*} \right] \quad (35)$$

$$r_1 + r_2 + r_3 + r_4 = 2\rho + (\nu + R^*)\phi_\alpha + f_k^* - \rho\phi_\gamma \quad (36)$$

where $\kappa \equiv \tilde{c}^* \nu \sigma \rho f_k^{*2} > 0$ is a constant.

When B has no eigenvalues with zero real parts, the steady state x^* is a hyperbolic fixed point for which the Hartman-Grobman Theorem and the Stable Manifold Theorem for non-linear systems hold. As a result, the asymptotic behavior of solutions near x^* – and hence its stability type – are determined by the linearization (32). Having this, and given that the government avoids global indeterminacies by proclaiming its 'exogenous' targets, we can discuss the determination of real variables near the steady-state equilibrium:

Definition 4 *The equilibrium displays real determinacy if there exists a unique solution to*

$$\dot{x}_t = B \times (x_t - x^*).$$

Given that (k_t, a_t) are predetermined, Proposition 5 follows directly from equation (35):

Proposition 5 *The steady state x^* is hyperbolic if and only if $\phi_\alpha \left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1 - \varphi^*} \right] \neq$*

0. Furthermore, a necessary condition for real determinacy is that

$$\phi_\alpha \left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1 - \varphi^*} \right] < 0 \quad \text{where} \quad \phi_\beta > 0 \quad (37)$$

Proposition 5 states that fiscal policies that exhibit $\left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1 - \varphi^*} \right] > 0$ must interact with a passive monetary policy. Note that $\frac{d \ln \left[\frac{1 - \tau(k_t, a_t)}{1 + \frac{1}{\nu} R(\pi_t)} f'(k_t) \right]}{d \ln f(k_t)} \approx -\frac{1}{1 - \tau^*} \phi_\beta$. That

is, the evolution of after-tax marginal product of capital along an equilibrium trajectory is sensitive to 'endogenous tax actions'. When fiscal policy exhibits responses to output such that $\phi_\beta < 0$, the after-tax marginal product of capital becomes positively associated with output and such policies induce multiple equilibria. The intuition is as follows: start from a steady state equilibrium, and suppose that the future return on capital is expected to increase. Indeterminacy cannot occur without distorting taxes, since a higher capital stock is associated with a lower rate of return under constant returns to scale. However, a feedback income-tax rule that exhibits $\phi_\beta < 0$ causes the after-tax return on capital to rise even further, thus validating agents' expectations, and any such trajectory is consistent with equilibrium. By contrast, a stance such that $\phi_\beta > 0$ reduces higher anticipated returns on capital from belief-driven expansions, thus preventing expectations from becoming self-fulfilling.

4 Determinacy Bounds for Interest and Tax Actions

In this section I provide a complete and analytical parameter boundary characterization for the distortionary tax rule. This characterization, a case often deferred to numerical analysis in the literature because of the complex interactions introduced by distortionary taxation, is accomplished here to the best of my knowledge for the first time. Leeper and Nason (2008) also consider the effects of distortionary taxation in a production economy. They show that the active-passive interaction carries over to the distortionary tax economy. Bhattarai et al. (2013) characterize analytically the parameter boundaries of policy coefficients in a New Keynesian model with a Taylor rule and a lump-sum tax response to debt and output. In addition Bhattarai et al. (2012) and Traum and Yang (2011) characterize numerically how parameters affect the passive monetary/active fiscal policy boundaries in larger New Keynesian models. These works show that parameter boundaries can depend on policy targets and steady-state policy values, as well as which endogenous variables the policy instruments are allowed to respond to. In what follows I conduct a comprehensive analysis of the boundary characterization to discover that unlike previous literature, where there are two distinct regions of the parameter subspace that deliver a unique rational expectations

equilibrium, our model economy delivers three distinct regions. Previous results show that in the regime M (a Taylor rule that interacts with passive fiscal policy), the monetary authority responds to inflation deviations from its target level sufficiently to stabilize the inflation path, while the fiscal authority adjusts government spending or tax policy to stabilize government debt growth. In regime F (a passive monetary rule that interacts with active fiscal policy), the fiscal authority does not take sufficient measures to stabilize debt; instead, the monetary authority pursues actions to stabilize debt growth through price adjustments. In contrast to previous literature, our framework gives rise to a new regime that is absent from models with lump-sum taxation. In this regime, a plausible range of Ricardian fiscal policies that interact with a passive monetary policy deliver a unique rational expectations equilibrium. The size of this range is determined by the rate of self-financing of tax cuts in the steady state, and is therefore indirectly linked to the choice of 'exogenous targets'.

4.1 Determinacy Bounds in Ricardian Regimes - An Analytical Approach

Consider fiscal rules that exhibit $\phi_\gamma > 1$. We know by now that all fiscal rules must exhibit $\phi_\beta > 0$, and we also have from Proposition 5 that the condition $\phi_\alpha \left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1 - \varphi^*} \right] < 0$ must always hold. It then follows immediately that if the rate of self-financing exceeds 1, monetary policy must be passive. Proposition 5 indicates that in economies that reside on the "wrong side" of their Laffer curve, monetary policy must be passive regardless of the fiscal stance. Consider now economies that reside on the "right side" of their Laffer curve. In such economies, a passive monetary policy must interact with fiscal policies such that $\phi_\gamma \in (1, \overline{\phi_\gamma})$ where $\overline{\phi_\gamma} = \frac{\phi_\beta}{-(\mathcal{RSF}^* - 1) \frac{\tau^*}{1 - \varphi^*}}$. Note that the measure of the interval $(1, \overline{\phi_\gamma})$ increases as the rate of self financing approaches 1. Formally, in the limit, $\overline{\phi_\gamma} \xrightarrow[\mathcal{RSF}^* \uparrow 1]{} +\infty$, which indicates that the range of tax-rate responses to government debt that must interact with passive monetary rules grows to infinity as the rate of self-financing approaches 1 from below. Note that increases in the rate of self-financing stem from two different sources. A simplification of equation (34) reads $\mathcal{RSF}^* = (1 - \varphi^*) \left(1 + \frac{R^*}{\tau^* \nu} \right)$. The first set of brackets contains the familiar form for rates of self-financing associated with the elasticity of tax rev-

enues. The second set of brackets exposes an additional source of increments to the rate of self-financing that is original in our model and stems from the monetary system. Permanent decreases in the velocity of money considerably increase the government's ability to levy inflation taxes. In addition, and independently of the structure of the banking system, rates of self-financing increase with the long-run rate of nominal interest. As the long-run nominal interest rate equals the inflation target plus the rate of time preference, this implicitly shows that rates of self-financing depend on the 'exogenous' inflation target. The upshot is that higher inflation in the long run induces higher rates of self-financing.

4.2 Determinacy Bounds in Non-Ricardian Regimes - An Analytical Approach

Proposition 5 provides necessary conditions for real determinacy. In order to obtain sufficient conditions for real determinacy I focus, in what follows, on a baseline regime for which it is possible to obtain sufficient conditions. I then perturb the baseline regime so as to approximate the general case. Consider a fiscal rule that exhibits $\phi_\gamma = 0$. In this regime, the income-tax rate does not respond to deviations of debt from its long-run level. Substituting $\gamma = 0$ into eq. (32), the system becomes

$$\dot{x}_t = B_{[\gamma=0]} \times (x_t - x^*) \quad (38)$$

where

$$B_{[\gamma=0]} = \begin{bmatrix} \widehat{B}_1 & \emptyset \\ \widehat{B}_2 & \rho \end{bmatrix}$$

and where \widehat{B}_1 is the upper left 3×3 submatrix of B , \widehat{B}_2 is the 1×3 vector $(B_{4,1}, B_{4,2}, B_{4,3})$, and \emptyset is a 3×1 vector of zeros. Examining $B_{[\gamma=0]}$, the dynamics of (c, π, k) are independent of government liabilities. This feature has two implications: (a) one eigenvalue of the (c, π, k, a) system is $\rho > 0$; (b) the remaining three eigenvalues are determined by \widehat{B}_1 so that the dynamics of (c, π, k) are completely determined by \widehat{B}_1 . It is straightforward to show that the three remaining eigenvalues satisfy

$$r_1 r_2 r_3 = -\kappa \phi_\alpha \phi_\beta \quad (39)$$

$$r_1 + r_2 + r_3 = \rho + (\nu + R^*) \phi_\alpha + f_k^* \quad (40)$$

which leads us to the following proposition:

Proposition 6 *If fiscal policy targets only output, then a sufficient condition for real determinacy is that monetary policy exhibits $\alpha < \frac{1}{1 + \frac{\rho}{\nu + R^*} + \frac{\rho + \delta}{\nu(1 - \tau^*)}} < 1$ and fiscal policy exhibits $\phi_\beta > 0$.*

Proposition 7 *Consider a regime that induces real determinacy near a hyperbolic steady state x^* where fiscal policy targets only output. Then, perturbations to γ in the neighborhood of $\bar{\gamma} = 0$ do not change the phase portrait of x^* as long as γ is not perturbed until its bifurcation point. Specifically, given $\bar{\alpha} < \frac{1}{1 + \frac{\rho}{\nu + R^*} + \frac{\rho + \delta}{\nu(1 - \tau^*)}}$ and $\phi_{\bar{\beta}} > 0$, any regime that exhibits $\phi_\gamma > 0$ will also induce a locally determinate equilibrium as long as the multiple of eigenvalues in the perturbed system does not change signs.*

Corollary 2 *Perturbations to γ around the policy stance $(\alpha, \beta, \gamma) = (\bar{\alpha}, \bar{\beta}, 0)$ induce real determinacy as long as the fiscal rule induces $-y^* d \ln y_t (\mathcal{RSF}^* - 1) \frac{\tau^*}{1 - \varphi^*} < \rho a^* d \ln a_t$. Hence it is non-Ricardian.*

The intuition of this result is as follows. Interactions between monetary and fiscal policies near $(\bar{\alpha}, \bar{\beta}, 0)$ continue to induce real determinacy if they comply with the following three principles. First, in all regimes, monetary policy is passive. Second, in all regimes tax-rate responses to output must exhibit $\phi_\beta > 0$. Finally, any deviation from the baseline regime $(\bar{\alpha}, \bar{\beta}, 0)$ must satisfy inequality (37). It is straightforward to show that (I) $\frac{d \ln \left[\frac{1 - \tau(k_t, a_t)}{1 + \frac{1}{2} R(\pi_t)} f'(k_t) \right]}{d \ln a_t} \approx -\frac{1}{1 - \tau^*} \gamma$ and that (II) $\frac{d \ln \left[\frac{1 - \tau(k_t, a_t)}{1 + \frac{1}{2} R(\pi_t)} f'(k_t) \right]}{d \ln f(k_t)} \approx -\frac{1}{1 - \tau^*} \phi_{\bar{\beta}}$. Also, given $\bar{\alpha}$, and hence that monetary policy is passive, inequality (37) implies that (III) $\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1 - \varphi^*} > 0$. Finally, note that as long as γ is non-negative we have that (IV) $\text{Sign}[d \ln a_t] = \text{Sign}[d \ln f(k_t)]$. Altogether, we obtain corollary 2 by substituting (I) and (II) into (III). This result shows that in order to induce real

determinacy, fiscal rules that let government debt grow at a rate greater than the addition to tax revenues must interact with passive monetary rules.

4.3 The Phase Portrait

To obtain determinacy bounds for practical uses, the model is calibrated at an annual frequency to the structural parameters of the US and EU economies. The annual (subjective) rate of time preference and the elasticity of intertemporal substitution are set according to the general consensus. We calibrate the elasticity of production technology so as to induce a steady-state Laffer curve that peaks at the levels obtained by Trabandt and Uhlig (2011). Specifically, we choose ϵ so as to induce maximal capital tax rates of 0.63 and 0.48 for the US and EU-14 economies, respectively. With reference to Trabandt and Uhlig's (2011) results, we calibrate the model so as to bring about elasticities of tax revenues of 0.5 and 0.2 for the US and EU economies, respectively. We set money velocity in the steady state so as to correspond to the US M2 and the EU M2 money velocities in October 2013.⁹ An overview of the calibration is provided in Table 1:

Table 1 - Structural parameters and Calibrations

Parameter	Description	US	EU
ρ	Subjective rate of time preference (% , annual)	2	2
σ	Elasticity of intertemporal substitution	0.5	0.5
δ	Rate of capital depreciation (% , annual)	7	7
β	Tax response to output	1	1
$1 - \epsilon$	Maximal tax rate (%)	63	48
φ^*	Elasticity of tax revenues	0.5	0.2
ν	M2 money velocity	1.57	0.97
\tilde{a}^*	Debt/GDP	1	0.93
π^*	Inflation target (%)	2	2
$\tau^* - \tilde{T}^*$	Surplus/GDP (% , implied)	-0.55	-2.26

Figure 1 enters here

Figure 2 enters here

Figures 1 and 2 show the phase portraits of the US and EU economies, respectively. Figures

⁹Sources: (I) Federal Reserve Bank of St. Louis - Velocity of M2 Money Stock, Ratio, Quarterly, Seasonally Adjusted. (II) Eurostat and ECB calculations.

1.1 and 2.1 show phase portraits where the income-tax rate is set to zero and the rate of self-financing is 1. When the model is calibrated so as to mimic lump-sum taxation, it yields determinacy bounds that coincide with those of Leeper (1991) and its subsequent literature. In this case, two demarcation lines, a horizontal line at $\alpha = 1$ and a vertical line at $\gamma = \rho\tilde{a}^*$, divide the parameter space into four areas that correspond to four regimes. The model reestablishes Leeper's (1991) well-known result that where taxes are lump-sum, two regimes bring about real determinacy. The first regime, corresponding to regime M, is where a Taylor rule interacts with Ricardian fiscal rules. The second regime, corresponding to regime F, is where a passive monetary rule interacts with non-Ricardian fiscal rules. As we know, both regime M and regime F are not without limitations. Benhabib et al. (2001, 2002) show that regime M is prone to liquidity traps. Davig et al. (2011) show that regime F generates a burst of inflation that devalues the existing nominal debt, thus placing upward pressure on inflation expectations and posing a substantial challenge to a central bank pursuing an inflation target. The distribution of outcomes for the path of future inflation in regime F has a fat right tail, revealing that only a small set of outcomes imply inflationary scenarios. Avoiding these scenarios requires the fiscal authority to renege on some share of either its liabilities or its future promised transfers.

Note, however, that distortionary taxation brings about determinacy bounds that are different from the conventional result. This result is demonstrated by Figures 1.2-1.3 and Figures 2.2-2.3. In these figures a new regime emerges that is neither regime M nor regime F. The 'new' regime includes an interaction of Ricardian fiscal policy with a passive monetary rule and appears in the figures as the light-grey area to the right of the vertical demarcation line $\gamma = \rho\tilde{a}^*$. This area takes a non-negligible size in the parameter space. Specifically, for the EU economy (Figure 2.2) it resides in most of the plausible range of fiscal responses. Figures 1.3 and 2.3 show the effect of embracing a lower debt/GDP targets for the two economies. The 'exogenous' change in the debt/GDP target has no effect on the slope of the income-tax Laffer curve near the new steady state, nor on rates of self-financing.¹⁰

¹⁰However, reductions in the debt/GDP target may have some effect on the size of the 'new' regime because it affects local dynamics, see eq. (41) in the appendix.

5 Dynamic Scoring under Various Regimes

In what follows we study the dynamic effects of various tax actions in two dimensions. We disentangle the effects of 'exogenous tax actions' from those of 'endogenous tax actions'. We start by examining transition paths to a steady state with a lower debt/GDP target, assuming no change in the long-run levels of inflation and income tax. We compare the transition paths and consequent welfare gains under three regimes, F, M, and New. We then examine the dynamic effects of tax reductions in the long run.

Consider an economy that resides at a steady state x^* corresponding to the calibration provided in Table 1. Let $U^* = \frac{1}{\rho}u(c^*)$ denote a measure of the representative household's welfare in this steady state. We measure welfare gains due to a transition from x^* to $x^{**} \equiv (c^{**}, \pi^{**}, k^{**}, a^{**}, \tau^{**}, T^{**}, R^{**})$ as $\widehat{U} \equiv \frac{U_0 - U^*}{U^*}$, where U_0 measures welfare according to eq. (1) along the (unique) trajectory that goes to x^{**} . It is straightforward to show that $\widehat{U} \simeq \frac{u'(c^*)c^*}{u(c^*)}\rho \int_0^{\infty} e^{-\rho t} \frac{c_t - c^*}{c^*} dt$, where $[c_t]_{t=0}^{+\infty}$ is the time path of consumption along the equilibrium trajectory, and the elasticity of the instantaneous utility, $\frac{u'(c^*)c^*}{u(c^*)} \leq 1$, is a constant. Hence, welfare gains are directly linked to the present value of the percentage deviation of consumption from its level prior to the transition. Accordingly, we rank various policy scenarios by their respective measures of \widehat{U} . We start by experimenting with policy scenarios designed to reduce the level of debt/GDP from 1 to 0.6 without changing the long-run targets of neither inflation nor the income-tax.

Figure 3 Enters here

Figure 4 enters here

Figures 3 and 4 show that in both economies social welfare is highest in the new regime, where Ricardian fiscal rules interact with a passive monetary rule. In regimes M and F welfare gains are negligibly positive or negative, whereas the new regime delivers positive welfare gains. Notice Figures 3.1 and 4.1, which portray the deficiencies of regime M. In these scenarios the US and EU economies practically reside in a liquidity trap. Even after a period of 30 years under regime M, output, inflation, and the tax rate are well below their respective 'exogenous' targets, and for output and inflation, relentless reductions in the nominal interest

rate will only accelerate this trend. Figures 3.1 and 4.1 show the dynamics that Benhabib et al. (2001, 2002) warn against. In fact, assuming stronger responses to inflation would yield dynamics that lead to zero nominal interest rates and multiple equilibria, thus mimicking exactly the situation described by Benhabib et al. (2001, 2002). Now notice Figures 3.3 and 4.3, which portray the deficiencies of regime F. In both economies, reducing the debt/GDP target while adhering to low responses of the tax rate to debt is inconsistent with rational expectations as long as the government does not renege on any portion of its obligations.

Figures 3.2 and 4.2 portray the merits of the new regime. They show how a successful regime can spur the economy to the desired steady state while avoiding both sovereign default and liquidity traps and inducing welfare gains. Initially, 'exogenous tax actions' increase the income-tax rate in response to government debt. The negative effect induced by the tax hike is entirely offset by monetary policy. The passive response of the nominal interest rate to inflation causes a reduction in the real rate of interest, thus motivating households to convert wealth to consumption. All in all, output and consumption return to their long-run levels, but the persistent increase in consumption has a positive effect on welfare.

5.1 Dynamic Scoring in Ricardian Regimes

Reductions in the tax-rate target reduce the present value of future surpluses, which may be inconsistent with the intention to ensure fiscal solvency. One way of financing the tax reduction is by increasing the inflation target. Scenario 1 in Table 2 corresponds to the policy experiment whose implications are provided by Figures 3 and 4. Scenarios 2 and 3 replicate the time paths of consumption and output as well as the overall welfare gains achieved in scenario 1. However, they allow different time paths for inflation and taxes as long as wealth effects are strong enough to cancel out substitution effects. The mechanism is to increase the 'exogenous' inflation target only as much as needed so as to leave the rate of self-financing equal to its level under scenario 1. We found that any 1-percent reduction to the 'exogenous' tax-rate target must be met with an increase of 3 percent and 1.8 percent to the 'exogenous' inflation target in the US and the EU, respectively, so as to stabilize the rate of self-financing near the new targets. An overview of the policy experiments under the new regime is provided in Table 2. The upshots of these experiments are provided in Figures

5 and 6.

Table 2 - Scenarios of dynamic scoring in a Ricardian regime

	Scenario 1		Scenario 2		Scenario 3	
Welfare gains (%)	0.98	0.28	0.98	0.28	0.98	0.28
Target	US	EU	US	EU	US	EU
\tilde{a}^*	0.6	0.6	0.6	0.6	0.6	0.6
$\Delta\tau^*(\%)$	0	0	-1	-1	-2	-2
$\pi^*(\%)$	2	2	5	3.8	8	5.6
$\tau^* - \tilde{T}^*(\%, \text{implied})$	-1.35	-2.92	-3.25	-4.73	-5.15	-6.54

Figure 5 enters here

Figure 6 enters here

5.2 Dynamic Scoring in Non-Ricardian Regimes

Non-Ricardian fiscal policies are not without limitations. From a policy maker's viewpoint, such policies can increase the risk of sovereign default. Note that Proposition 4 implies that in a non-Ricardian regime the real value of financial wealth must jump at $t=0$ so as to bring about market equilibrium. Formally, this means that instead of an initial condition for financial wealth we have a boundary condition stating how financial wealth must evolve along an equilibrium trajectory.¹¹ Solving the boundary condition yields the usual result that in equilibrium, the value of financial wealth equals the present value of future surpluses. This poses yet another complication: to compute the time path of surpluses we must know the initial value of all the state variables, including that of financial wealth. Thus, the initial value of financial wealth depends on the entire equilibrium, which depends on the initial value of financial wealth. This implies that the initial value of financial wealth and the entire trajectory must be determined simultaneously. We obtain the initial value of government debt in a non-Ricardian regime as the limit to a Krasnoselski-Mann-Bailey sequence.¹² Table

¹¹Equation (24) specifies the boundary condition.

¹²See the technical appendix.

3 describes scenarios where increments to the 'exogenous' inflation target are enough to offset two adverse effects on the real value of government debt. The first derives from the non-Ricardian nature of regime F. Once the 'exogenous' debt/GDP target is reduced and tax rates are not expected to respond to the new target, an alternative source of revenue must be obtained. We obtain that given a mute response of tax rates to debt, an increase in the 'exogenous' inflation target can increase the flow of seigniorage revenues, thus stabilizing the value of government debt. This effect is obtained in scenario 4. Secondly, any decrease in the 'exogenous tax rate target' must be met with an adequate increase in the 'exogenous inflation target' so as to stabilize the rate of self-financing of tax cuts near the new steady state. This mechanism is obtained in scenarios 5 and 6. An overview of the policy experiments under regime F is provided in Table 3. The upshots of these experiments are provided in Figures 7 and 8.

Table 3 - Scenarios of dynamic scoring a non-Ricardian regime

	Scenario 4		Scenario 5		Scenario 6	
Welfare gains (%)	-1.28	-0.54	-1.28	-0.54	-1.28	-0.54
Target	US	EU	US	EU	US	EU
\tilde{a}^*	0.6	0.6	0.6	0.6	0.6	0.6
$\Delta\tau^*(\%)$	0	0	-1	-1	-2	-2
$\pi^*(\%)$	8.6	2.95	11.6	4.7	14.8	6.5
$\tau^* - \tilde{T}^*(\%, \text{ implied})$	-5.55	-3.9	-7.45	-5.7	-9.5	-7.5

Figure 7 enters here

Figure 8 enters here

6 Concluding Remarks

This paper formalizes the consequences of combined changes in the level of income taxes and the nominal rate of interest designed so as to achieve long-run levels of public debt and output. We augment the Ramsey model to include distortionary taxation and liquidity constraints. As a result, changes in taxes influence output. We construct a dynamic setup that gets around complications associated with dynamic Laffer curves. In this setup, tax cuts are self-financing at rates higher than 1 minus the slope of the Laffer curve because of a wedge brought about by the monetary system. Furthermore, economies with relatively low money velocity are bound to exhibit rates of self-financing that are in general less sensitive to tax changes. We also show that global indeterminacies can be avoided. Global determinacy is conditional on the availability of common knowledge with respect to long-run targets and a very particular policy regime that emerges only under distortionary taxation. Calibrations of the model to the US and EU economies show that exogenous tax actions devised so as to return public debt to its historical level yield welfare gains even though output may fall in the short run. In contrast, tax cuts designed to increase output in the long run may trigger a default, and therefore must be offset with higher exogenous inflation so as to stabilize the rate at which planned tax cuts are self-financing. All in all, our model indicates that this set of policies ends up producing negligible welfare gains.

Promising routes for extending the analysis include investigating the importance of New Keynesian elements to the characteristics of tax changes. An important addition to the model, I think, would be the inclusion of long-term debt in the arsenal of public finance instruments. This augmentation has the potential to alter some of the results.

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Technical Appendix

Preliminaries

Definition (the index of a fixed point) [Hirsch and Smale (1976)] *Let $\bar{x} \in \mathbb{R}^n$ be a hyperbolic equilibrium, that is, the eigenvalues of $Df(\bar{x})$ have nonzero real parts. In this case, the index $ind(\bar{x})$ of \bar{x} is the number of eigenvalues (counting multiplicities) of $Df(\bar{x})$ having negative real parts.*

The Stable Manifold Theorem [Guckenheimer and Holmes (1983) Theorem 1.3.2]

Suppose that $\dot{x} = f(x)$ has hyperbolic fixed point \bar{x} . Then there exist local stable and unstable manifolds $W_{loc}^s(\bar{x}), W_{loc}^u(\bar{x})$, of the same dimensions n_s, n_u as those of the eigenspaces E^s, E^u of the linearized system, respectively, and tangent to E^s, E^u at \bar{x} . $W_{loc}^s(\bar{x}), W_{loc}^u(\bar{x})$ are as smooth as the function f .

The Hartman-Grobman Theorem [Guckenheimer and Holmes (1983) Theorem 1.3.1]

If $Df(\bar{x})$ has no zero or purely imaginary eigenvalues, then there is a homeomorphism h defined on some neighborhood U of \bar{x} in \mathbb{R}^n locally taking orbits of the nonlinear flow Φ_t of $\dot{x} = f(x)$ to those of the linear flow $e^{tDf(\bar{x})}$ of $\dot{y} = Df(\bar{x})y$. The homeomorphism preserves the sense of orbits and can also be chosen to preserve parametrization by time.

The index of a hyperbolic fixed point is the dimension of the stable manifold. In the context of our model, and given that we have two predetermined variables, equilibrium \bar{x} is determinate if and only if $ind(\bar{x}) = 2$. The implications for our model appear in Tables A.1 and A.2 below, where rr_i denotes the real part of eigenvalue $r_{i \dots i=1, \dots, 4}$.

Table A.1: Index and equilibria in a four-dimensional vector space with two predetermined variables

Sign(rr_1)	Sign(rr_2)	Sign(rr_3)	Sign(rr_4)	det(A)	Trace(A)	Index	Equilibrium
+	+	+	+	> 0	> 0	0	no-equilibrium
-	+	+	+	< 0	≥ 0	1	no-equilibrium
-	-	+	+	> 0	≥ 0	2	unique
-	-	-	+	< 0	≥ 0	3	multiple
-	-	-	-	> 0	< 0	4	multiple

Table A.2: Index and equilibria in a three-dimensional vector space with two predetermined variables

Sign(rr_1)	Sign(rr_2)	Sign(rr_3)	det(\hat{A})	tr(\hat{A})	Index	Equilibrium
+	+	+	> 0	> 0	0	no-equilibrium
-	+	+	< 0	≥ 0	1	no-equilibrium
-	-	+	> 0	≥ 0	2	unique
-	-	-	< 0	< 0	3	multiple

A linear approximation to eq. (21)-(24) near a hyperbolic fixed point A linear approximation near the steady state reads

$$\begin{aligned}
 \dot{x}_t &= B \times (x_t - x^*) & (41) \\
 B &\equiv \begin{bmatrix} 0 & -\frac{\sigma\tilde{c}^*(\rho+\delta)}{\nu+R^*}f^* & -\frac{\sigma\tilde{c}^*}{1+\frac{1}{\nu}R^*}f_k^{*2}\left(\beta+\frac{\tau^*}{1-\varphi^*}\right) & -\gamma\frac{\sigma(\rho+\delta)}{1-\tau^*}\frac{\tilde{c}^*}{\tilde{a}^*} \\ 0 & \rho+\delta+(\nu+R^*)\frac{\alpha-1}{\alpha} & \frac{\nu}{\alpha}\frac{f_k^{*2}}{f^*}\left(\beta+\frac{\tau^*}{1-\varphi^*}\right) & \gamma\frac{\nu}{\alpha}\frac{f_k^*}{f^*}\frac{1}{\tilde{a}^*} \\ -1 & 0 & f_k^*-\delta & 0 \\ 0 & \alpha f^*\left[\frac{\alpha-1}{\alpha}\tilde{a}^*-\frac{1}{\nu}\right] & -f_k^*\left[\frac{1}{\nu}R^*+\beta+\tau^*\right] & \rho-\frac{\gamma}{\tilde{a}^*} \end{bmatrix} \\
 x_t &\equiv \begin{bmatrix} c_t \\ \pi_t \\ k_t \\ a_t \end{bmatrix} & x^* &\equiv \begin{bmatrix} c^* \\ \pi^* \\ k^* \\ a^* \end{bmatrix}.
 \end{aligned}$$

(Asterisks denote steady-state levels; f_k^* , f^* , \tilde{a}^* , \tilde{c}^* are marginal product of capital, GDP, debt-to-GDP, and consumption-to-GDP, respectively; and k_t , a_t , are predetermined state variables.)

We obtain analytically that the determinant of B is

$$- [\tilde{c}^* \nu \sigma \rho f_k^{*2}] \frac{\alpha-1}{\alpha} \left[\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho \tilde{a}^*} \left(\frac{1}{\nu} R^* - \frac{\tau^* \varphi^*}{1-\varphi^*} \right) \right],$$

and that the trace of B is

$$2\rho + (\nu + R^*) \frac{\alpha-1}{\alpha} + (\rho + \delta) \frac{1 + \frac{1}{\nu} R^*}{1-\tau^*} - \frac{\gamma}{\tilde{a}^*}.$$

Proof of Proposition 5 Assume that $\phi_\alpha \left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1-\varphi^*} \right] \neq 0$. Then the multiple of eigenvalues is nonzero, which indicates that there is no zero eigenvalue. Assume now that $\phi_\alpha \left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1-\varphi^*} \right] = 0$. Then either $\phi_\alpha = 0$ or $\left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1-\varphi^*} \right] \neq 0$. In what follows I show that either policy induces a zero eigenvalue, i.e. that there is a bifurcation at $\alpha = 1$ and given ϕ_β there is a bifurcation at $\gamma = \rho \tilde{a}^* \frac{\phi_\beta}{-(\mathcal{RSF}^* - 1) \frac{\tau^*}{1-\varphi^*}}$. Note that implementing $\alpha = 1$ and $\left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1-\varphi^*} \right] = 0$ simultaneously brings about a codimension two bifurcation.

Assume $\phi_\alpha = 0$ and $\left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1-\varphi^*} \right] \neq 0$.

Substituting $\phi_\alpha = 0$ into equation (41) we obtain that

$$B_{[\phi_\alpha=0]} \equiv \begin{bmatrix} 0 & -\frac{\sigma \alpha \tilde{c}^* (\rho + \delta)}{\nu + R^*} f^* & -\frac{\sigma \tilde{c}^*}{1 + \frac{1}{\nu} R^*} f_k^{*2} \left(\beta + \frac{\tau^*}{1-\varphi^*} \right) & -\gamma \frac{\sigma (\rho + \delta)}{1-\tau^*} \frac{\tilde{c}^*}{\tilde{a}^*} \\ 0 & \rho + \delta & \frac{\nu}{\alpha} \frac{f_k^{*2}}{f^*} \left(\beta + \frac{\tau^*}{1-\varphi^*} \right) & \gamma \frac{\nu}{\alpha} \frac{f_k^*}{f^*} \frac{1}{\tilde{a}^*} \\ B_{3,1} & 0 & B_{3,3} & 0 \\ 0 & B_{4,2} & B_{4,3} & B_{4,4} \end{bmatrix}$$

where $B_{i,j}$ $i, j = 1, \dots, 4$ are components of B specified in eq. (41), respectively. Where $\alpha = 1$ the first row is a multiplication of the second row by $-\frac{\sigma \alpha \tilde{c}^*}{\nu + R^*} f^*$. Consequently $B_{[\phi_\alpha=0]}$ is singular.

Assume $\phi_\alpha \neq 0$ and $\left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1-\varphi^*} \right] = 0$.

Substituting $\tilde{\gamma} \equiv \rho \tilde{a}^* \frac{\phi_\beta}{-(\mathcal{RSF}^* - 1) \frac{\tau^*}{1-\varphi^*}}$ into equation (41) we obtain that

$$B_{[\tilde{\gamma}]} \equiv \begin{bmatrix} 0 & B_{1,2} & B_{1,3} & \psi B_{1,3} \\ 0 & B_{2,2} & B_{2,3} & \psi B_{2,3} \\ B_{3,1} & 0 & B_{3,3} & 0 \\ 0 & B_{4,2} & B_{4,3} & \psi B_{4,3} \end{bmatrix}$$

where $\psi \equiv \frac{\rho}{-f_k^*(\mathcal{RSF}^*-1)\frac{\tau^*}{1-\varphi^*}}$ is a constant. It is straightforward to notice that the determinant of $B_{[\bar{\gamma}]}$ equals zero. Thus, a monetary-fiscal regime such that $\phi_\alpha \left[\phi_\beta + \phi_\gamma (\mathcal{RSF}^* - 1) \frac{\tau^*}{1-\varphi^*} \right] = 0$ brings about a non hyperbolic equilibrium. This concludes the first part of the proof.

The second part is straightforward. From Table A.1 it follows that a necessary condition for equilibrium determinacy is $\det(B) > 0$. The proof of the proposition is concluded by requiring that the right-hand side of equation (35) is positive.

Proof of Proposition 6 Equilibrium is determinate only where $\text{ind}(\bar{x}) = 2$. $B_{[\gamma=0]}$ is block recursive with one positive eigenvalue at the lower right 1×1 submatrix, and so we obtain the dimension of the stable manifold only by examining \widehat{B}_1 . Observe Table A.2. $\det(\widehat{B}_1) > 0$ is a necessary condition. Furthermore, we must rule out the case where $\text{ind}(\bar{x}) = 0$ by requiring that policy also induce $\text{tr}(\widehat{B}_1) < 0$. To conclude, we can ensure that $\text{ind}(\bar{x}) = 2$ by implementing a policy that brings about $r_1 r_2 r_3 > 0$ and $r_1 + r_2 + r_3 < 0$.

It follows from equation (39) that $r_1 r_2 r_3 > 0 \Leftrightarrow \phi_\alpha \phi_\beta < 0$. So $\det(\widehat{B}_1) > 0$ under two regimes: $\{\phi_\alpha > 0 \text{ and } \phi_\beta < 0\}$ or $\{\phi_\alpha < 0 \text{ and } \phi_\beta > 0\}$. We rule out the first regime, first, because $\phi_\beta < 0$, and second, because we obtain from equation (40) that in this regime $\text{tr}(\widehat{B}_1) > 0$, and we cannot rule out the possibility of No-equilibrium. By contrast, in the second regime, $\phi_\alpha < 0$ and hence $\alpha < 1$. We can ensure that $\text{tr}(\widehat{B}_1) < 0$ by requiring that $\alpha < \frac{1}{1 + \frac{\rho}{\nu + R^*} + \frac{\rho + \delta}{\nu(1 - \tau^*)}} < 1$. *QED.*

Proof of Proposition 7

Preliminaries

Theorem 1 [Hirsch and Smale (1976) Chap.16] *Let $f : W \rightarrow E$ be a C^1 vector field and $\bar{x} \in W$ an equilibrium of $\dot{x} = f(x)$ such that $Df(\bar{x}) \in L(E)$ is invertible. Then there exists a neighborhood $U \subset W$ of \bar{x} and a neighborhood $\mathfrak{R} \subset \mathcal{U}(W)$ of f such that for any $g \in \mathfrak{R}$ there is a unique equilibrium $\bar{y} \in U$ of $\dot{y} = g(y)$. Moreover, if E is normed, for any $\epsilon > 0$ we can choose \mathfrak{R} so that $|\bar{y} - \bar{x}| < \epsilon$.*

Theorem 2 [Hirsch and Smale (1976) Chap.16] *Suppose that \bar{x} is a hyperbolic equilibrium. In Theorem 1, then, \mathfrak{R} and U can be chosen so that if $g \in \mathfrak{R}$, the unique equilibrium $\bar{y} \in U$ of $\dot{y} = g(y)$ is hyperbolic and has the same index as \bar{x} .*

Proof Consider now complex fiscal rules that exhibit $\gamma \neq 0$. In what follows I show that for small perturbations of γ near $\gamma = 0$ the system is structurally stable. Consider the system $\dot{x}_t = g_{[\gamma]}(x_t)$ where $\gamma = 0 + \varepsilon$, $\varepsilon > 0$. Then a linearization reads

$$\dot{x}_t = [B_{[\gamma=0]} + \varepsilon\Delta] \times (x_t - x^*) \quad (42)$$

where

$$\Delta \equiv \begin{bmatrix} 0 & 0 & 0 & -\frac{\sigma(\rho+\delta)}{1-\tau^*} \frac{\tilde{c}^*}{\tilde{a}^*} \\ 0 & 0 & 0 & \frac{\nu}{\alpha} \frac{f_k^*}{f^*} \frac{1}{\tilde{a}^*} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\tilde{a}^*} \end{bmatrix}.$$

Since $B_{[\gamma=0]}$ is invertible, by implicit function theorem, $\dot{x}_t = g_{[\gamma]}(x_t)$ continues to have a unique solution $x^{**} = x^* + O(\varepsilon)$ near x^* for sufficiently small ε . Moreover, since we restrict attention to a set of policies that satisfy proposition 4, we ensure that $B_{[\gamma=0]} + \varepsilon\Delta$ is invertible, which implies that $x^{**} = x^*$ is the unique solution to equation (42). Furthermore, since the matrix of the linearized system $B_{[\gamma=0]} + \varepsilon\Delta$ has eigenvalues that depend continuously on ε , no eigenvalues can cross the imaginary axis if ε remains small with respect to the magnitude of the real parts of the eigenvalues of $B_{[\gamma=0]}$. Thus, the perturbed system (42) has a unique fixed point with eigenspaces and invariant manifolds of the same dimensions as those of the unperturbed system, and with an ε that is close in position and slope to the unperturbed manifolds. The main idea of this proposition is that perturbations are in the parameter space $\{\gamma\}$. By construction such perturbations do not change the steady state itself. However, they may change the phase portrait of the steady state. Thus, starting from a determinate equilibrium, as long as γ does not reach its bifurcation point, the phase portrait of the (unchanged) steady state should not be affected by the perturbation. The

formal proof of Corollary 4 follows directly from the following Theorems. Specifically, in the terminology of Theorems 1 and 2, I choose $\bar{y} = \bar{x}$ and $g(y)$ that differs from $f(x)$ up to the perturbation of γ .

Obtaining the initial value of financial wealth as a solution to a fixed point problem

In what follows we use the Krasnoselski-Mann-Bailey theorem to prove that the interaction between non-Ricardian fiscal policies and a passive monetary rule induces a unique determination of the initial real value of financial wealth and, as a result, a unique determination of the entire equilibrium trajectory. Let C be a closed convex subset of a Banach space E . A mapping T on C is called a non-expanding mapping if $\|T(x) - T(y)\| \leq \|x - y\|$ for all $x, y \in C$. We denote by $\{\bar{x}\}$ the set of fixed points of T . If C is compact, then Schauder's fixed-point theorem yields that $\{\bar{x}\}$ is non-empty. Obtaining members of $\{\bar{x}\}$ is then feasible via a Krasnoselski and Mann's type sequence. Krasnoselski's Theorem is the following result:

Krasnoselski's Theorem If C is a convex, bounded subset of a uniformly convex Banach space and if T is a mapping of C into a compact subset of C such that $\|T(x) - T(y)\| \leq \|x - y\|$, then the sequence obtained by choosing x_1 in C and defining $x_{n+1} = \frac{1}{2}[x_n + T(x_n)]$ converges to some z in C and $T(z) = z$.

Bailey (1974) offered a proof for the case in which C is a closed interval of the real line. Bailey's Theorem is the following:

Bailey's Theorem If T takes $[a, b]$ into itself and $\|T(x) - T(y)\| \leq \|x - y\|$, then the sequence obtained by choosing x_1 in $[a, b]$ and defining $x_{n+1} = \frac{1}{2}[x_n + T(x_n)]$ converges to some z in $[a, b]$ and $T(z) = z$.

Mann's (1953) theorem states that if T is a continuous function that takes $[a, b]$ into itself and has a unique fixed point, z in $[a, b]$, then the Krasnoselski-Bailey sequence converges to z for all choices of x_1 in $[a, b]$.

We are now at the point where we can calculate valuations of government debt when the government adopts non-Ricardian stances. The main idea is to recognize that given the initial stock of capital, the monetary rule, and the fiscal rule, the present value of future surpluses depends only on the initial value of government debt. The latter measure thus becomes a mapping from a closed interval on the real line to itself. Once the dimensionality is one, it is straightforward to obtain the initial value of financial wealth as a limit to a Krasnoselski-Mann-Bailey sequence. Corresponding to the autonomous system $\dot{x}_t = g(x_t)$, specified in Proposition 1 and where $g : W \rightarrow E$, there are maps $\Phi : \Omega \rightarrow W$ where $(t, c, \pi, k, a, \tau, T, R) \in \Omega$ satisfying $\dot{\Phi}_t = g(\Phi_t)$. In view of definition 1, Φ_t is defined by letting $\Phi_t(c, \pi, k, a, \tau, T, R) \equiv \Phi(t, c, \pi, k, a, \tau, T, R)$ be a solution curve sending 0 to (k_0, a_0) and sending $+\infty$ to $(c^*, \pi^*, k^*, a^*, \tau^*, T^*, R^*)$. Propositions 6 and 7 provide conditions that guarantee the existence of a unique homeomorphism Φ_t on $[0, +\infty)$.

Let $\Psi \equiv \int_0^{\infty} Q_s X_s ds$ indicate the present value of future primary surpluses. Then $\Psi : W \rightarrow \mathfrak{R}_+$ corresponds to the mapping of the flow Φ_t into a positive real number. Let F denote the composition $\Psi \circ \Phi$. Then $F : \Omega \rightarrow \mathfrak{R}_+$, and $\tilde{F}(a_t) \equiv F(c_t, \pi_t, k_t, a_t, \tau^*, T^*, R^*)$ returns the valuation of government liabilities given that the tax rate target is τ^* , the inflation target is π^* , the debt/GDP target is \tilde{a}^* , the state variables are (a_t, k_t) , and (c_t, π_t) are jump variables. In particular, $\tilde{F} : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$ is a single-dimensional function that measures solely the effect of the state variable a_t on the valuation of government debt, taking as given the stock of capital k_t and the 'exogenous' targets. The economy is in equilibrium¹³ if and only if $\forall t \in [0, +\infty) \ a_t = \tilde{F}(a_t)$, and we can say that we pinned down the initial value of financial wealth if we obtain an a_0 that is a solution to the fixed-point problem $a_0 = \tilde{F}(a_0)$. A crucial feature of \tilde{F} stems from the fact that the economy resides in a non-Ricardian regime. In such regimes, increments to the real value of government debt are met by tax hikes that do not fully offset the increase in government debt, and therefore cause a less than one-for-one increase in the present value of future surpluses. The formal representation of this feature is $\frac{\partial \tilde{F}}{\partial a} < 1$, and this leads us to the following result:

¹³For example, in the steady state $a^* = \tilde{F}(a^*) = \frac{1}{\rho} [f(k^*)(\tau^* + \frac{1}{\nu}R^*) - \bar{T}]$.

Lemma 1 $\tilde{F}(a)$ is a non-expanding mapping.

Proof A Taylor expansion for $\tilde{F}(a)$ reads $d\tilde{F}(a) = \left[\frac{\partial F}{\partial a}\right]_{a^*} da + residual$. Taking norms

yields that

$$\|d\tilde{F}(a)\| \leq \left\| \left[\frac{\partial F}{\partial a}\right]_{a^*} \right\| \|da\| + \|residual\|$$

$$\Rightarrow \|d\tilde{F}(a)\| \leq \left\| \left[\frac{\partial F}{\partial a}\right]_{a^*} \right\| \|da\|$$

and since non-Ricardian regimes imply that $\frac{\partial \tilde{F}}{\partial a} < 1$ we obtain that $\|d\tilde{F}(a)\| \leq \|da\|$. *QED.*

Once we establish that $\tilde{F}(a)$ is a non-expanding mapping that maps a closed interval on the real line into itself, we can apply a Krasnoselski-Mann-Bailey sequence to obtain a solution to the fixed -point problem $a_0 = \tilde{F}(a_0)$.

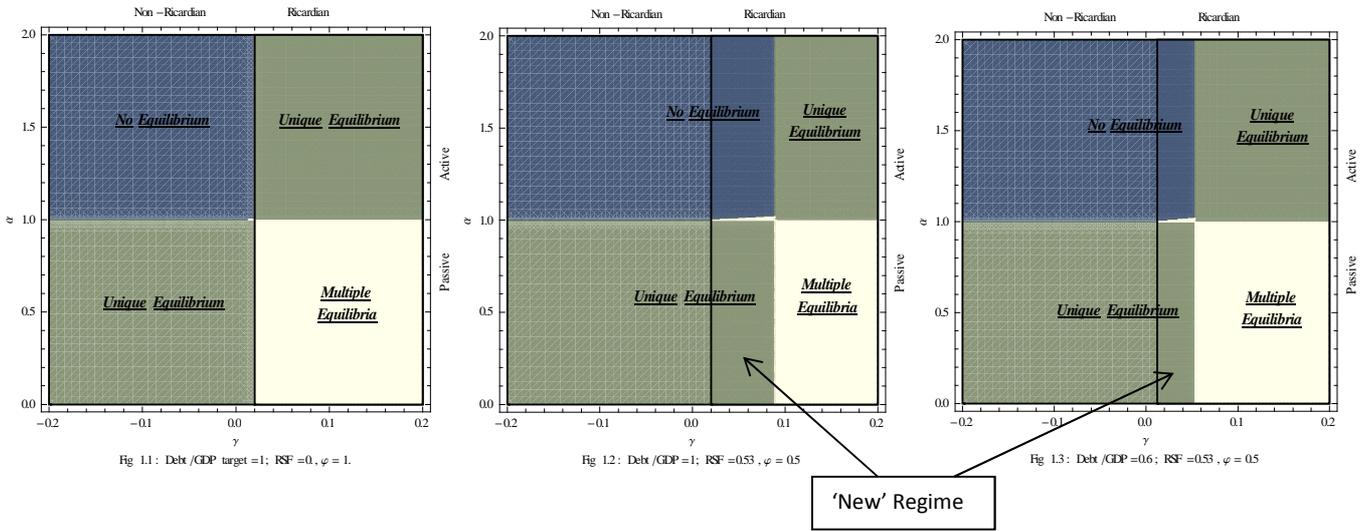


Figure 1: Phase portraits in the policy parameter space for the US economy.

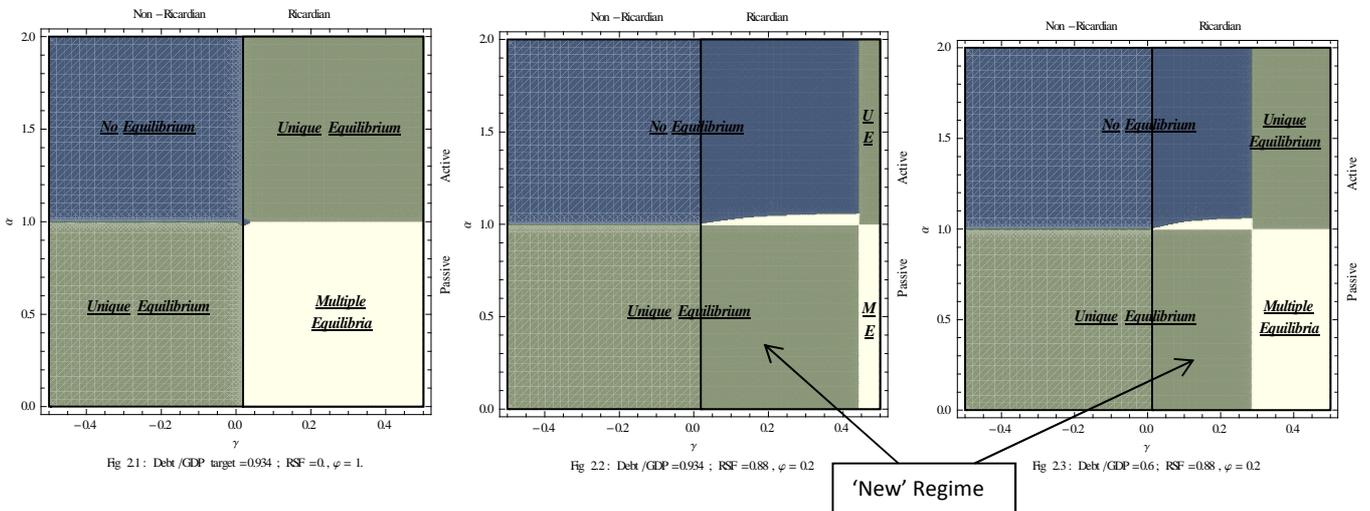


Figure 2: Phase portrait in the policy parameter space for the EU.

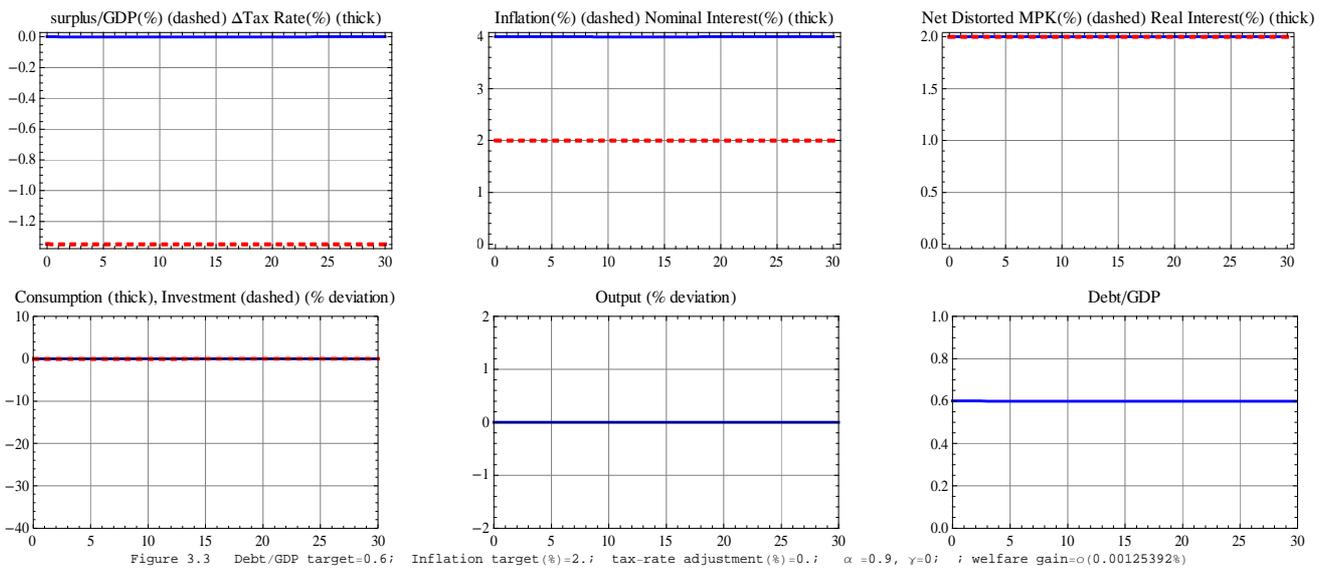
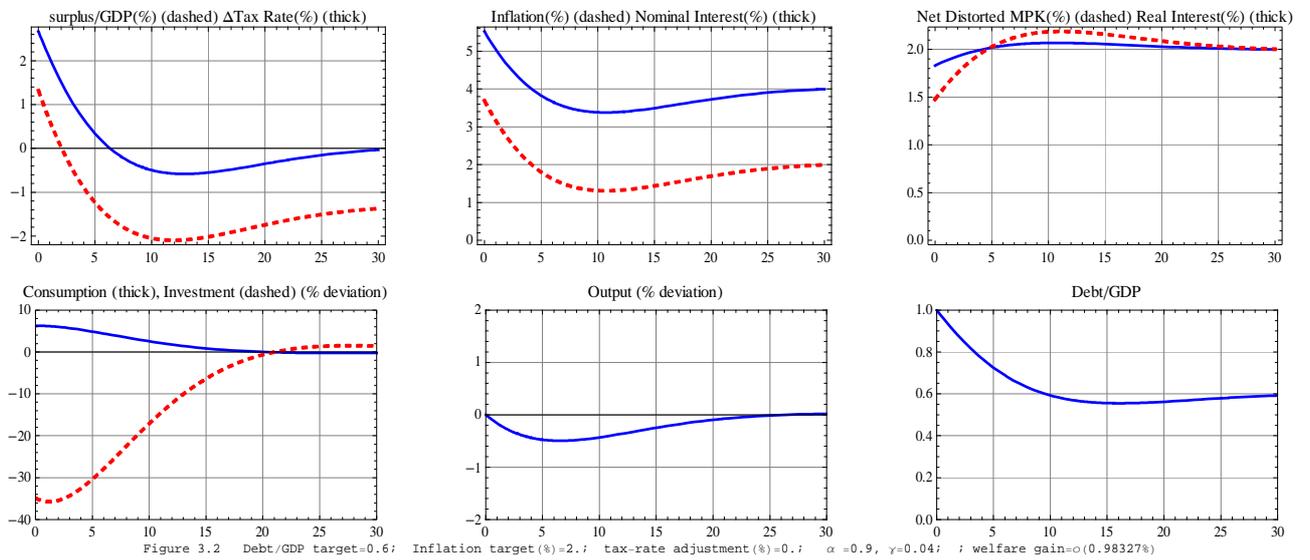
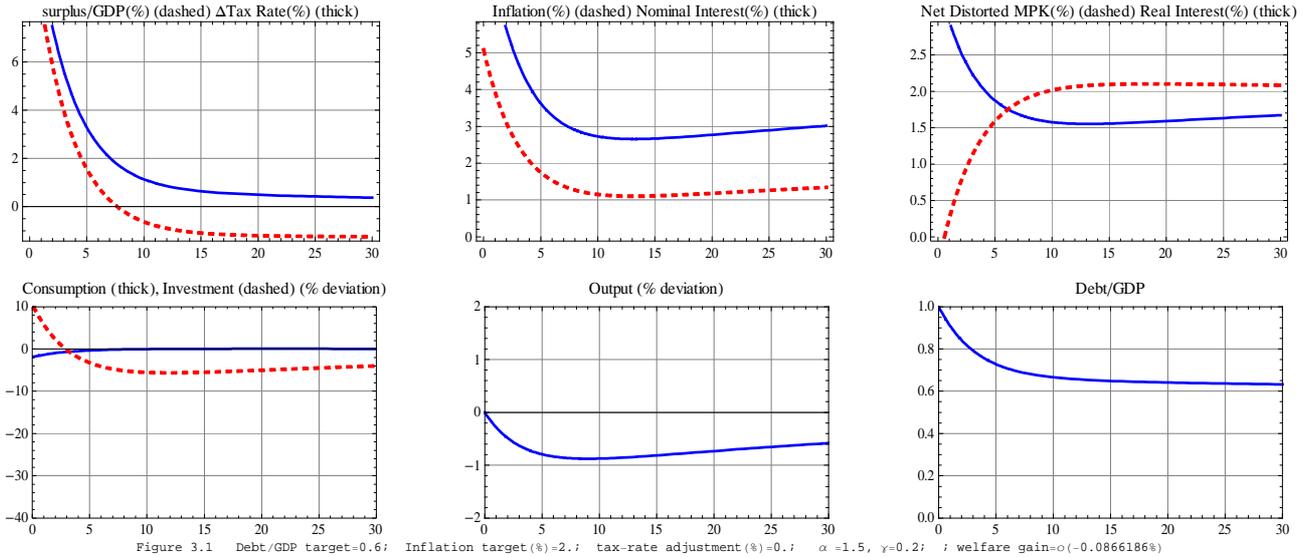


Figure 3: US economy - Transitional dynamics to Debt/GDP=0.6, $\Delta\pi^*=0, \Delta\tau^*=0$

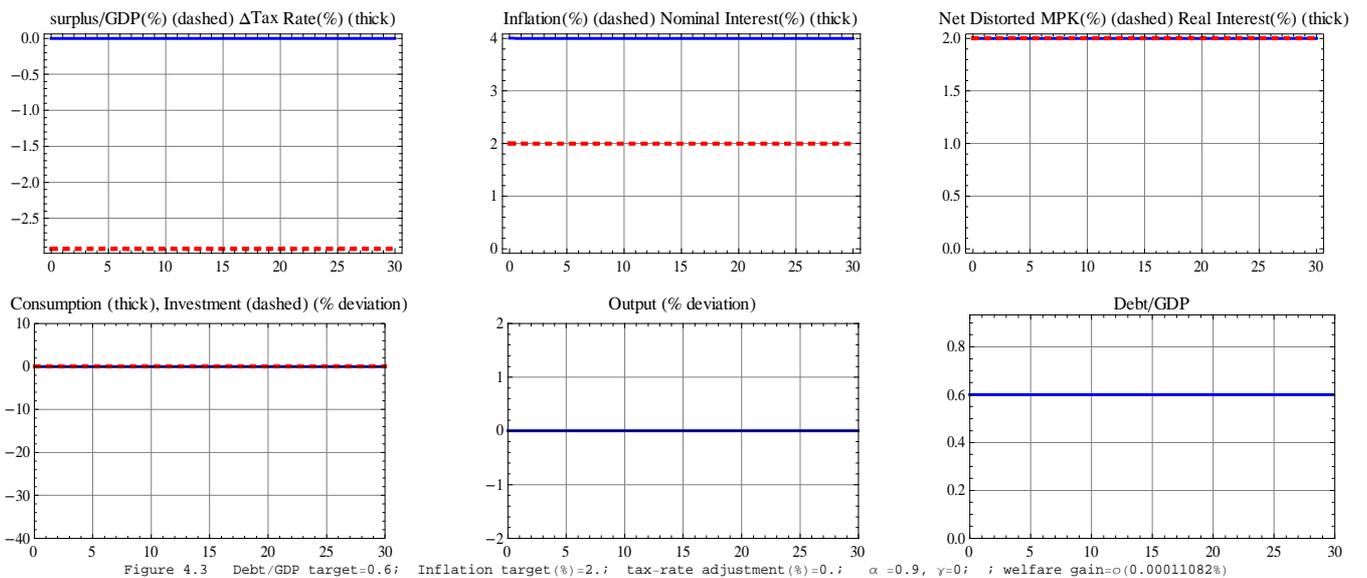
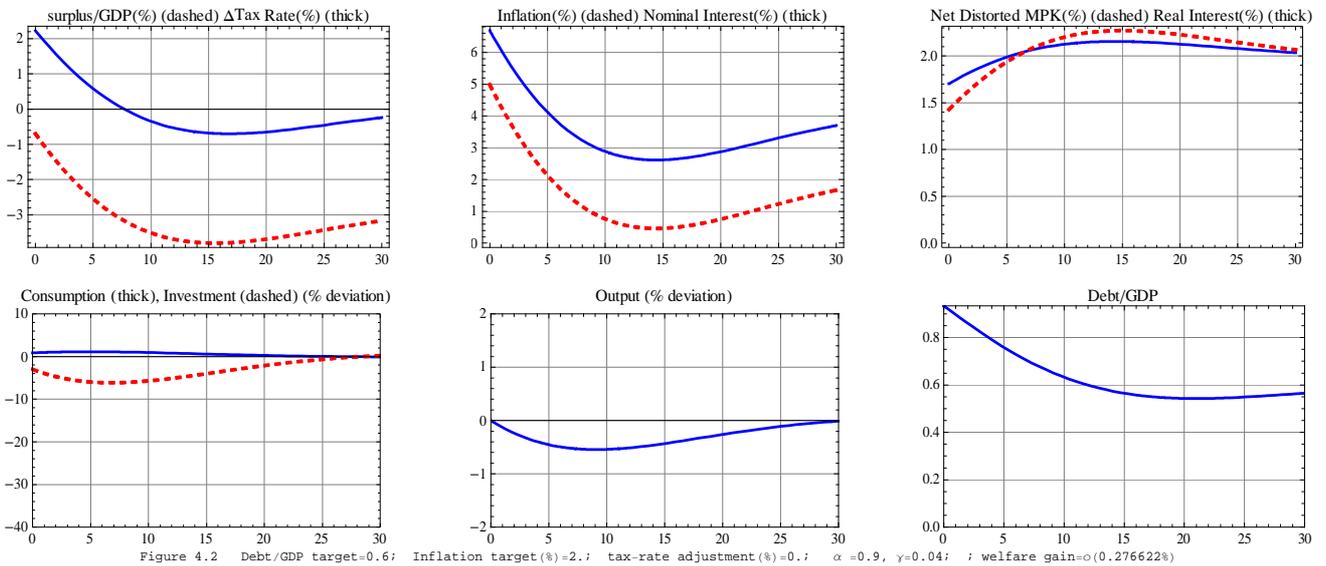
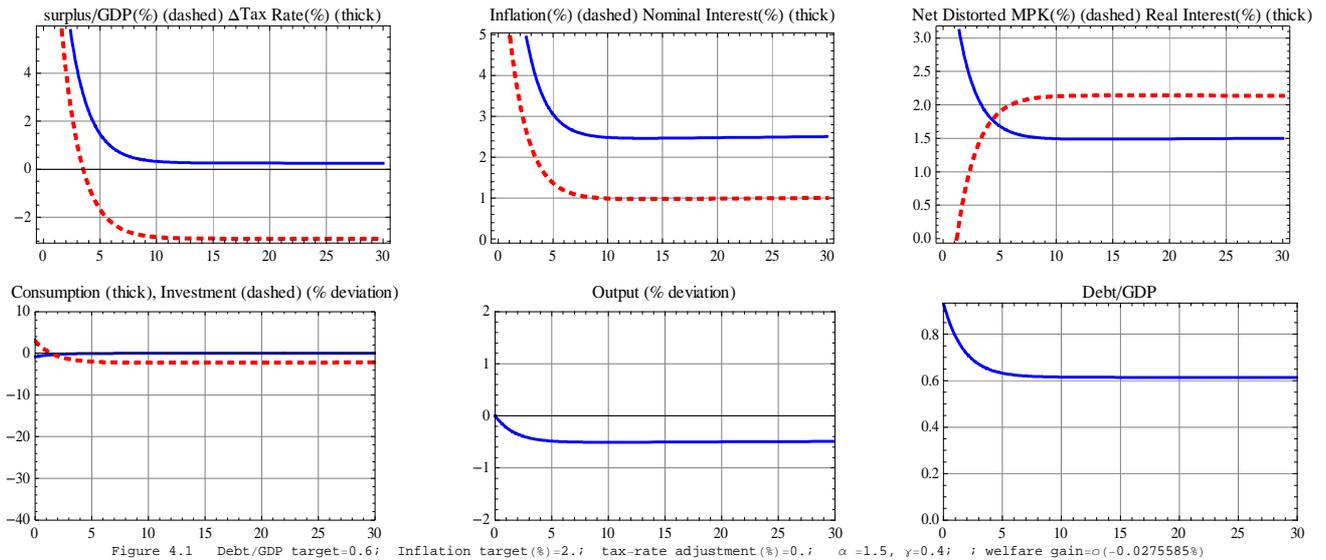


Figure 4: EU economy - Transitional dynamics to Debt/GDP=0.6, $\Delta\pi^*=0, \Delta\tau^*=0$

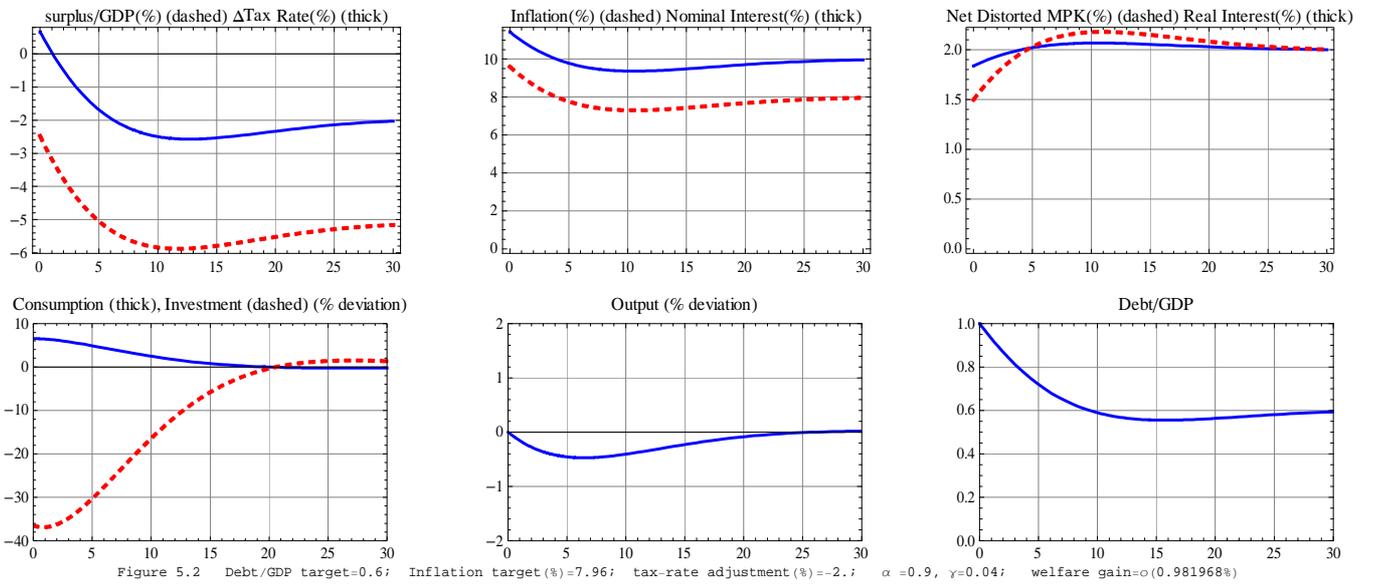
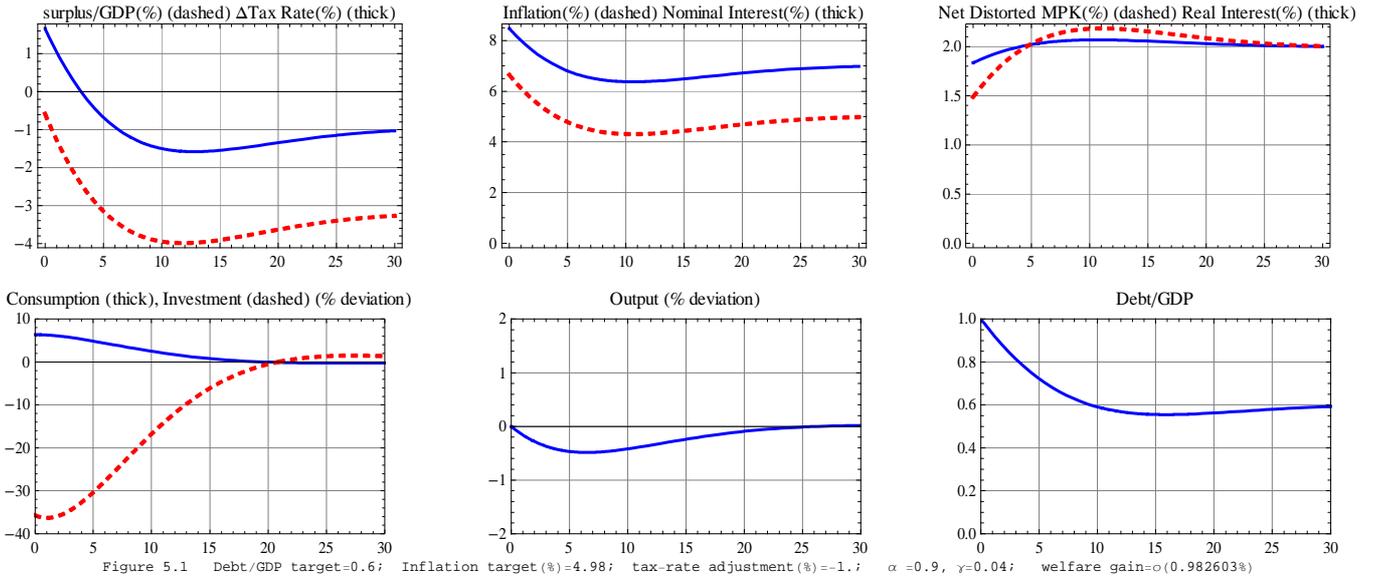


Figure 5: Avoiding Liquidity Traps in Ricardian Regimes – Calibration to the US economy

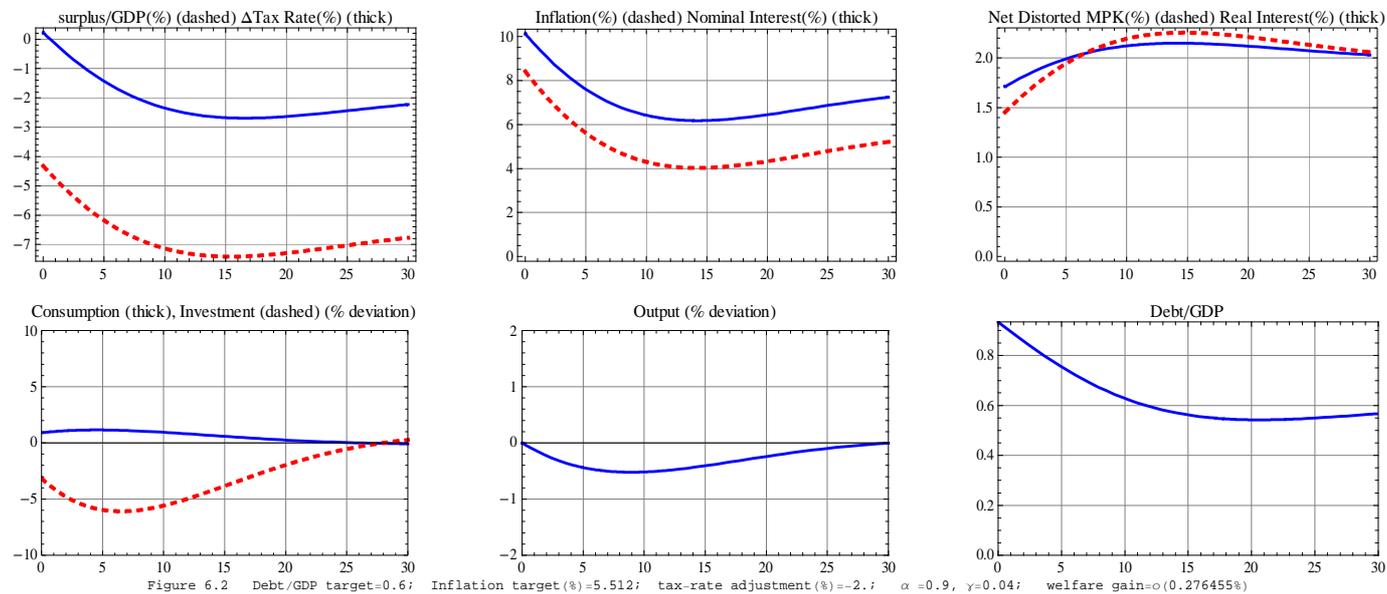
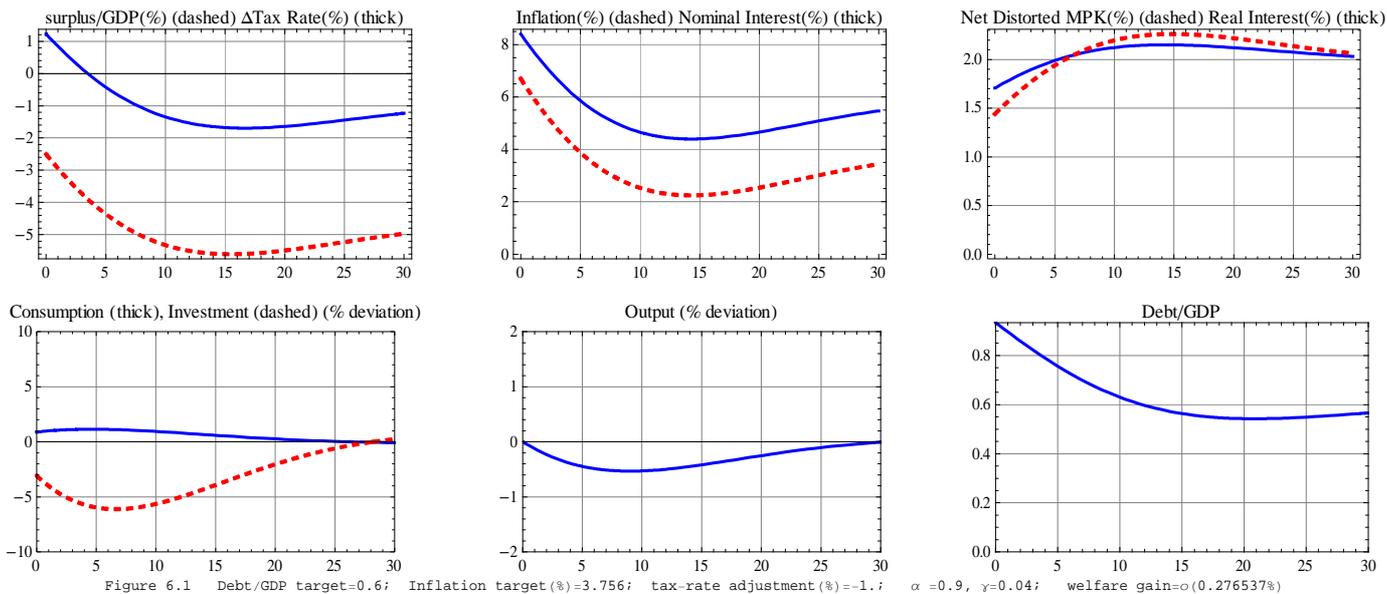


Figure 6: Avoiding Liquidity Traps in Ricardian Regimes – Calibration to the EU economy

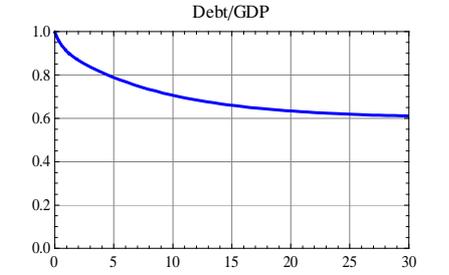
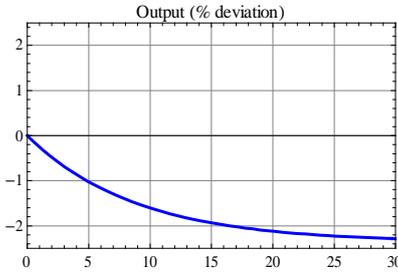
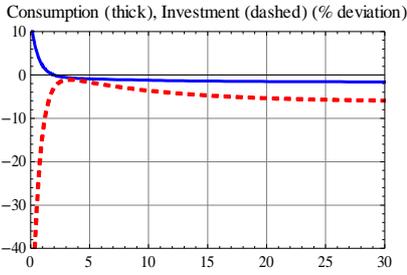
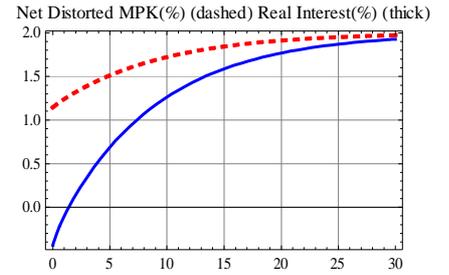
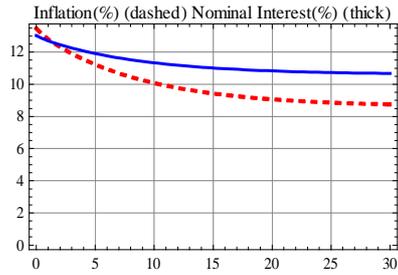
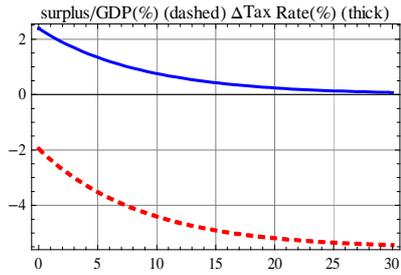


Figure 7.1 Debt/GDP target=0.6; Inflation target (%)=8.6; tax-rate adjustment (%)=0.; $\alpha = 0.5, \gamma = 0$; welfare gain=0(-1.28866%)

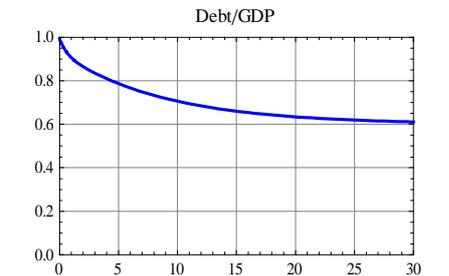
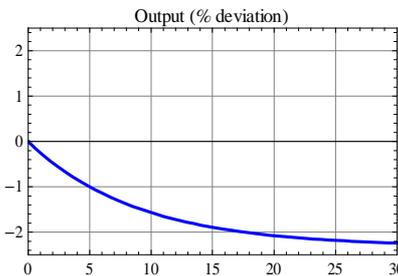
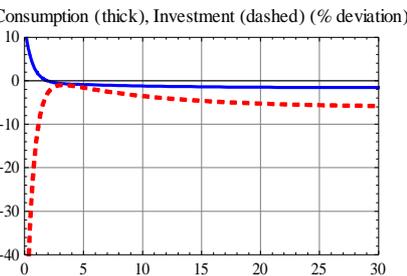
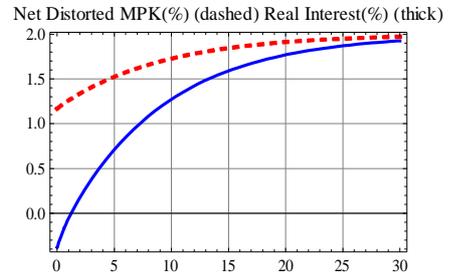
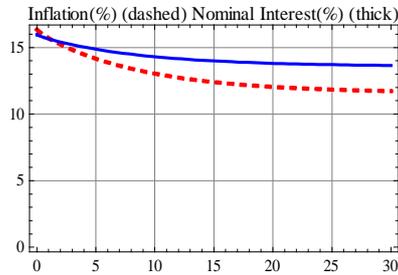
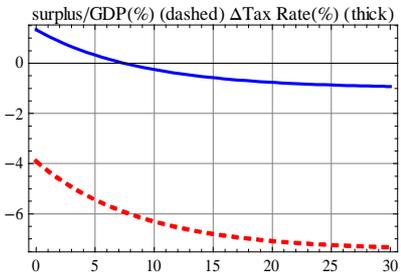


Figure 7.2 Debt/GDP target=0.6; Inflation target (%)=11.58; tax-rate adjustment (%)=-1.; $\alpha = 0.5, \gamma = 0$; welfare gain=0(-1.2648%)

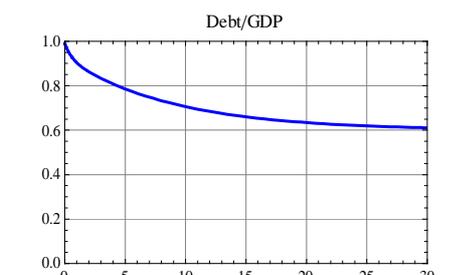
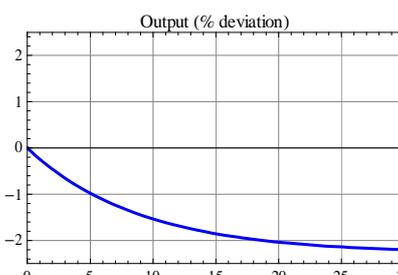
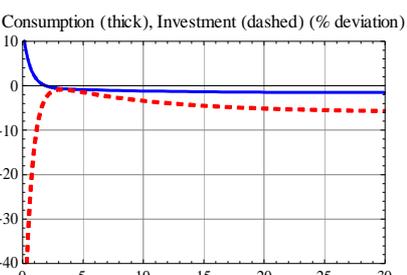
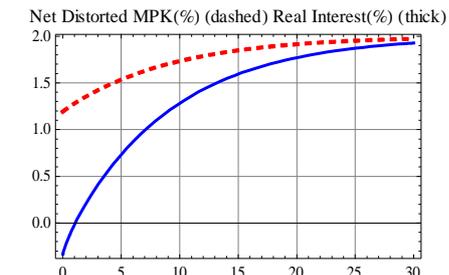
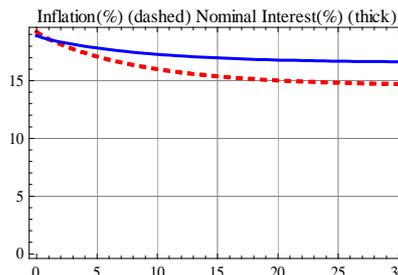
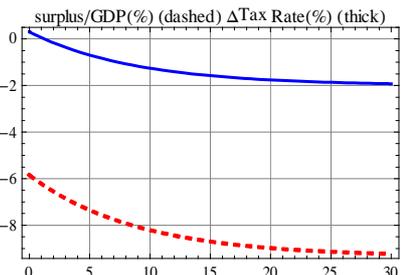


Figure 7.3 Debt/GDP target=0.6; Inflation target (%)=14.56; tax-rate adjustment (%)=-2.; $\alpha = 0.5, \gamma = 0$; welfare gain=0(-1.24204%)

Figure 7: Avoiding Sovereign Default in non-Ricardian Regimes – Calibration to the US economy

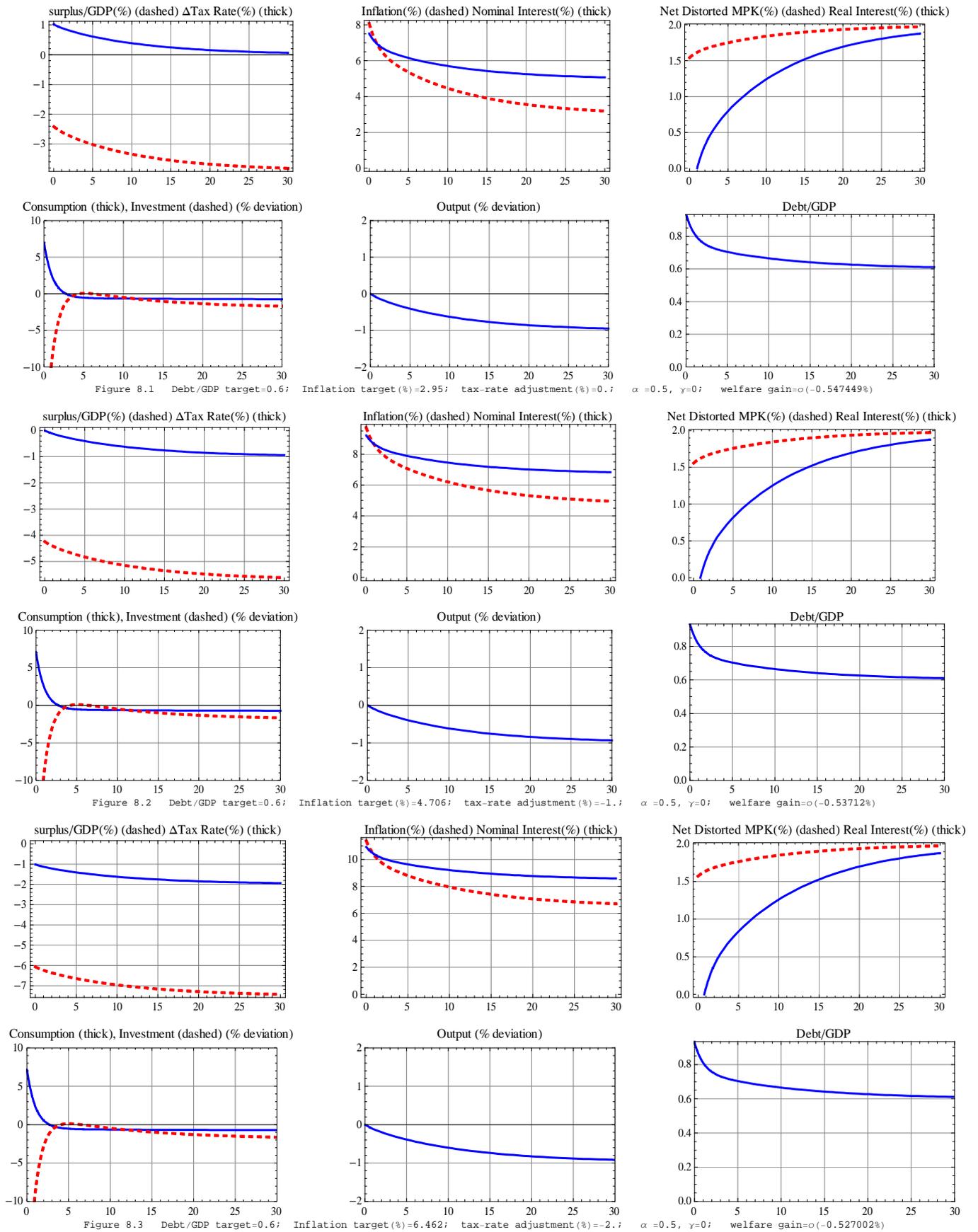


Figure 8: Avoiding Sovereign Default in non-Ricardian Regimes – Calibration to the EU economy