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Strategic and stable pollution with finite set of economic agents and a finite set of consumption commodities: a Pareto comparison

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Abstract In this paper, we apply the framework of a public bad economy with a finite set of economic agents and a finite set of consumption commodities. The pollution of an agent is emitted into the air while consuming his consumption goods. A linear public bad economy model with a single private good had been introduced in Shitovitz and Spiegel (Econ Theory 22(1):17–31, 2003) and here we reconsider its extension to finite number of private goods and prove existence of a core ‘trading’ allocation (Perets et al. J Math Econ 48(3):163–169, 2012) that Pareto dominates the Nash allocation. This mathematical model embodies the restriction of consumption by all polluting agents, to decrease the amount of the public bad, affecting the whole economy, worldwide. Note specifically that the Lindahl allocation may not Pareto dominate the Nash allocation in some finite economy, in contrast to well-known asymptotic results.

Keywords Game theory · Pollution · Core · Pareto Domination

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1 Introduction

This paper discusses the notion of a core ‘trading’ allocation (Perets et al. 2012) that Pareto dominates the Nash allocation in pure public ‘bad’ economies, introduced by Shitovitz and Spiegel (2003). The economy contains a finite set of economic agents who consume a finite number of consumption commodities; the consumption of these commodities produces the public bad which is ‘pollution’ that is emitted into the air and has a negative effect on all the agents in the economy.

Under the Nash non-cooperative concept, each consumer assumes that the pollution emitted by all other consumers is not affected by her own emission. Under this assumption, extending (Perets et al. 2012), we prove the existence of the non-cooperative Nash equilibrium in the current paper for a model with linear pollution, with a finite number of private goods and one public bad.

In a seminal paper, Champsaur (1975) provided a proof that the core of an economy with one private good and one public good is a vN&M stable set. Based on this result and Peleg (1986), Shitovitz and Spiegel (2001) proved that there exists a core allocation that Pareto dominates the inefficient Nash allocation, in the same model, with a finite number $n \geq 2$ of consumers. This result has been generalized in Perets et al. (2012) to such economy with a mixed measure space of consumers.

The main result of our paper states that, in such a general model, there exists a core allocation that Pareto dominates the unique Nash allocation. Our economic game model is specific for our analysis, to use the result of both Peleg (1986) and Demange (1987), that the core of an ordinal convex game is externally stable.

The paper is organized as follows: Section 2 presents economies with a finite number of private goods and a pure public bad with a finite number of consumers. In Sect. 3, we define the linear pollution model. In Sect. 4, we prove the existence of the Nash equilibrium in our pure public ‘bad’ economy. In Sect. 5, we define the core notion in our model and prove our main theorem (existence of core allocation which Pareto dominates the Nash allocation).

2 The pure public ‘bad’ economy

We reconsider an economy E with a finite set $N = \{1, 2, \dots, n\}$ of economic agents and K private commodities. The vectors $c^i \in R_+^K$ represent units of private commodities (goods) each agent $i \in N$ consumption is restricted to the amount given in the i 's initial endowment vector ω^i . We assume that $0 \leq c^i \leq \omega^i$ for all agents. Following the model of a public bad economy, let $G \geq 0$ represents units of the pure public bad commodity (e.g., pollution). The utility of each consumer $i \in N$ is given by the function $u^i(c^i, G)$ defined on the appropriate subset of R_+^{K+1} . It is assumed that agents derive positive marginal utility from the private commodities and negative marginal utility from the pure public bad. As a pure public commodity, the same quantity G is the ‘Pollution’ equally affecting all agents in the public bad economy. In the consumption of an agent, the by-product pollution units are emitted as a result. We assume that G ,

the pure public bad, is the sum of all units of pollution emitted irrespective of their source. Let $E = (n; \omega^1, \dots, \omega^n; u^1, \dots, u^n)$ denote this finite economy.

3 The model of a linear pollution economy

We consider the pure public bad economy E . We assume that each consumer i is initially endowed with a strictly positive amount $\omega_i \in R_{++}^K$ of the private goods. Each consumer decides how much to consume from her initial endowment, and hence, as a by-product, its level of pollution units emitted is assumed to be 'linear'. That is, the level of pollution $\hat{g}_i = \sum_{k=1}^K \alpha_k c_{i,k}$. Let $\hat{G} = \sum_{i \in N} \hat{g}_i$ be the level of the aggregate pollution emitted in the economy. By changing units and replacing utilities, we assume that $\alpha_k = 1$ for all $k = 1, \dots, K$.

Apply now our assumptions above, and let obtain after 'normalization' that the utilities are given by $u_i : [0, W]^K \times [0, W] \rightarrow R_+$ where $W \equiv \sum_{k=1}^K \sum_{i=1}^N \omega_{i,k}$. We assume that u_i are continuous functions and $u_i(c_i, G) = 0$ if either $c_i \in \partial R_+^K$ or $G = W$. Moreover, on $(0, W]^K \times [0, W)$, $u_i(c_i, G)$ are strictly quasi-concave and strictly monotonic increasing in c_i (the private goods), and strictly monotonic decreasing in G (the pollution).

4 Existence of Nash equilibrium in a pure public 'bad' economy

The Nash pollution level is characterized by the non-cooperative game theoretic notion of Nash equilibrium. When maximizing her utility, each consumer needs to choose her optimal consumption bundle $c_i^* \in R_+^K$ such that $0 \leq c_i^* \leq \omega^i$ so the Nash n -tuple strategy $s^* = (c_i^*)_{i \in N}$ constitutes Nash equilibrium in the non-cooperative game with utilities $U^i(c_1, \dots, c_n) = u^i(c^i, G)$ where $G = G_{-i}^* + \hat{g}_i = \sum_{j \neq i} \hat{g}_j^* + \hat{g}_i$ where $\hat{g}_i = \sum_{k=1}^K c_k^i$ and similarly $\hat{g}_j^* = \sum_{k=1}^K (c_k^j)^*$ for $j \neq i$. Note that by our assumptions on the utilities and the linear technology of pollution, it follows that all $U^i(c_1, \dots, c_n)$ are continuous in all variables and quasi-concave in the vector variable c^i over the desired strategy sets.

Proposition 1 *Our finite economy admits a unique Nash allocation. We denote it by $((c_i^*)_{i \in N}, G^*)$ where of course we have $G^* = \sum_{k=1}^K (\sum_{i \in N} c_{i,k}^*)$.*

Proof Follows immediately from Nash Theorem. $\square$

Remark 1 By our assumptions, the Nash allocation satisfies $((c_i^*)_{i \in N}, G^*) \in (0, W]^{K \cdot N} \times [0, W)$.

Remark 2 The Nash equilibrium is the most applied non-cooperative solution concept in game theory. But in general, the Nash allocation is not efficient. We proceed to the most cooperative solution concept, the core, which encompasses both efficiency and stability. We formalize a relationship between these solution concepts to answer positively this inefficiency problem for public bad economies, raised by many researchers (e.g., Coase 1960, Chichilnisky and Heal 1994). Note that in this finite framework of a

finite game' the Lindahl allocation may not Pareto dominate the Nash allocation, and therefore we need further properties of the core of our economy, to prove the existence of a core allocation which Pareto dominates the Nash allocation.

5 The core

The cooperative NTU game (N, V) of our public bad economy is given by: for the empty coalition we have $V(\emptyset) = \emptyset$. For any nonempty coalition $S \in 2^N$

$$V(S) = \left\{ u \in R_+^N : \begin{array}{l} \exists (c_i \in R_+^K)_{i \in S}, G \in R_+ \\ G = \sum_{i \in N \setminus S} \sum_{k=1}^K \omega_{i,k} + \sum_{i \in S} \sum_{k=1}^K c_{i,k} \\ c_i \leq \omega_i, \forall i \in S \\ \text{and } u^i \leq u^i(c^i, G), \forall i \in S \end{array} \right\}$$

Proposition 2 *The cooperative NTU game (N, V) of our public bad economy is an ordinal convex game. That is, $V(S_1) \cap V(S_2) \subseteq V(S_1 \cup S_2) \cup V(S_1 \cap S_2)$ for $S_1, S_2 \subseteq N$. (Note that in particular we have a stronger property: $V(S_1) \cap V(S_2) \subseteq V(S_1 \cup S_2)$.)*

Proof Assume $\tilde{u} \in V(S_1) \cap V(S_2)$. Then, there exist $(c'_i \in R_+^K)_{i \in S_1}, G' \in R_+$ and $(c''_i \in R_+^K)_{i \in S_2}, G'' \in R_+$, that satisfy $\tilde{u}^i \leq u^i(c'_i, G'), \forall i \in S_1$ with $\tilde{u}^i \leq u^i(c''_i, G''), \forall i \in S_2$. Assume w.l.o.g. that $G' \leq G''$. Define the consumption commodity bundles for $S_1 \cup S_2$ by $\tilde{c}_i = \begin{cases} c'_i & i \in S_1 \\ \omega_i & i \in S_2 \setminus S_1 \end{cases}$ and therefore $\tilde{G} \equiv G'$. Then:

$$\tilde{u}^i \leq u^i(\tilde{c}_i, G') = u^i(c'_i, G'), \forall i \in S_1,$$

and

$$\tilde{u}^i \leq u^i(c''_i, G'') \leq u^i(\tilde{c}_i, G') = u^i(\omega_i, G'), \forall i \in S_2 \setminus S_1.$$

Thus, $\tilde{u} \in V(S_1 \cup S_2)$. □

Remark 3 We want to apply the result of Peleg (1986) to the core of the NTU game, which states that the core of an ordinal convex game is a Von Neumann and Morgenstern (1944) solution. Indeed, the ordinal convex game (N, V) satisfies all the Peleg's conditions, (by the continuity and the monotonicity properties of the utilities, and Proposition 2). Thus, the core $C(N, V)$ is a Von Neumann–Morgenstern stable set and hence is externally stable.

Remark 4 To obtain our main result (Theorem 1 below), we assume now further assumptions on the utility functions:

(A.1.) $u_i(c_i, G)$ are continuously differentiable and have non-vanishing partial derivatives on $(0, W]^K \times [0, W)$.

(A.2.) The slope of the indifference curve with respect to $c_{i,k}$ and G , defined as: $MRS_{G-c_{i,k}}^i \equiv -\frac{u_{G,i}^i}{u_{c_{i,k},i}^i}(c_i, G)$ depends only on $(c_{i,k}, G)$, and is strongly monotonic increasing in $(c_{i,k}, G)$ on $(0, W]^K \times [0, W)$.

Remark 5 (A.1.) and (A.2.) under strict quasi-concave utilities are equivalent to $(c_{i,k})_{k=1,\dots,K}$ are strictly normal commodities and the pollution G is a strict inferior commodity (see [Perets et al. 2012](#) and Appendix A).

Remark 6 Note that under assumptions (A.1.) and (A.2.), our public bad economy admits a unique Nash equilibrium (see [Perets and Shitovitz \(2013\)](#) and Appendix B).

Theorem 1 *In our public bad economy, any Nash allocation is (weakly) Pareto dominated by some allocation in the core $C(N, V)$.*

Proof Let $(c_i^*, G^*)_{i \in N}$ be a Nash allocation, and $u_i^* \equiv u_i(c_i^*, G^*)$ for each $i \in N$. If $u^* \in C(N, V)$, then trivially the core allocation $((c_i^*)_{i \in N}, G^*)$ induces the Nash utilities. Otherwise, since the core is a Von Neumann–Morgenstern stable set, by external stability, there exist nonempty coalition $S \subseteq N$ and $\bar{u} \in C(N, V) \cap V(S)$ such that $\bar{u}_i > u_i^*$ for $i \in S$. By definition of $V(S)$, there exists allocation $((\bar{c}_i)_{i \in S}, \bar{G})$ which satisfies $\bar{u}_i \leq u_i(\bar{c}_i, \bar{G})$ for each $i \in S$. We prove now that $\bar{u} \geq u^*$.

Case 1 $\bar{G} \leq G^*$. Assume by negation that there exists a consumer $j \in N \setminus S$, for whom $\bar{u}_j < u_j^*$. Then, the coalition $S \cup \{j\}$, with allocation $((\bar{c}_i)_{i \in S}, \bar{G})$ and $(c_j^*, \bar{G})$ for consumer j blocks $\bar{u}$ via $S \cup \{j\}$. Indeed, we have $u_j(c_j^*, \bar{G}) \geq u_j(c_j^*, G^*) \equiv u_j^* > \bar{u}_j$, and for all $i \in S$ we have equality. As, by Remark 1, (c_j^*, G^*) is in $(0, W]^K \times [0, W)$, we obtain a contradiction to $\bar{u} \in C(N, V)$.

Case 2 $\bar{G} > G^*$. Since $u_i(\bar{c}_i, \bar{G}) > u_i(c_i^*, G^*)$ for all $i \in S$, then there exist $i \in S$ and $k \in K$ that satisfy $\bar{c}_{i,k} > c_{i,k}^*$ and by definition of $V(S)$ we have also $\bar{c}_{i,k} \leq \omega_{i,k}$. Therefore, $0 < c_{i,k}^* < \omega_{i,k}$. Thus, using the F.O.C. for the Nash maximization problem, of the interior solution $c_{i,k}^*$, we have: $\frac{\partial u_i}{\partial c_{i,k}}(c_i^*, G^*) + \frac{\partial u_i}{\partial G}(c_i^*, G^*) = 0$. Hence, $MRS_{G-c_{i,k}}^i(c_i^*, G^*) \equiv -\frac{u_{G,i}^i(c_i^*, G^*)}{u_{c_{i,k},i}^i(c_i^*, G^*)} = 1$. By (A.2.), $MRS_{G-c_{i,k}}^i$ depends only on $(c_{i,k}, G)$, and is strongly monotonic increasing in $(c_{i,k}, G)$ on $(0, W]^K \times [0, W)$, by $\bar{c}_{i,k} > c_{i,k}^*$ and $\bar{G} > G^*$ it follows that $MRS_{G-c_{i,k}}^i(\bar{c}_i, \bar{G}) > 1$. Therefore $\frac{\partial u_i}{\partial c_{i,k}}(\bar{c}_i, \bar{G}) + \frac{\partial u_i}{\partial G}(\bar{c}_i, \bar{G}) < 0$. Obviously, we have $(\bar{c}_i, \bar{G}) \in R_{++}^{K+1}$, therefore, it follows that there is $\varepsilon > 0$ such that $u_i(\bar{c}_i - \varepsilon e_k, \bar{G} - \varepsilon) > u_i(\bar{c}_i, \bar{G})$. Thus, S blocks $\bar{u}$ using $(\bar{c}_j, \bar{G} - \varepsilon)$ for $j \in S \setminus \{i\}$ and $(\bar{c}_i - \varepsilon e_k, \bar{G} - \varepsilon)$ for i . Contradicting $\bar{u} \in C(N, V)$. $\square$

Appendix A: Strict ordinal normality of the utilities and the normality properties of the commodities

(A.1.) and (A.2.) under strict quasi-concave utilities are equivalent to $(c_k)_{k=1,\dots,K}$ being strictly normal commodities and the pollution G is a strict inferior commodity.

Proof For C^2 , strict quasi-concave utility function $u(c, G)$, on $(0, W]^K \times [0, W)$, we have that the marginal rate of substitution between G and c_k is denoted by $MRS_{G-c_k} \equiv -\frac{u_G}{u_{c_k}}(c, G)$ and depends only on (c_k, G) .

1. Assume first that (A.1.) and (A.2.) hold, that is: $\frac{\partial MRS_{G-c_k}}{\partial c_k} = -\frac{u_{c_k G} \cdot u_{c_k} - u_{c_k c_k} \cdot u_G}{(u_{c_k})^2} > 0$ and $\frac{\partial MRS_{G-c_k}}{\partial G} = -\frac{u_{GG} \cdot u_{c_k} - u_{c_k G} \cdot u_G}{(u_{c_k})^2} > 0$. Then, it follows that $u_{c_k G} \cdot u_{c_k} - u_{c_k c_k} \cdot u_G < 0$ and $u_{GG} \cdot u_{c_k} - u_{c_k G} \cdot u_G < 0$, for all $k = 1, \dots, K$ on $(0, W]^K \times [0, W)$. Since $I > 0$ and $(p_1, \dots, p_K) > 0$ it follows that the F.O.C. for the consumer maximization problem for $(\hat{c}_i, \hat{G}) \in R_{++}^{K+1}$ is $\frac{P_G}{P_{c_k}} = \frac{u_G(\hat{c}_1(I), \dots, \hat{c}_K(I), \hat{G}(I))}{u_{c_k}(\hat{c}_1(I), \dots, \hat{c}_K(I), \hat{G}(I))}$. Assuming the function is differentiable at $((\hat{c}_i)_{i \in N}, \hat{G}) \in (0, W]^K \times [0, W)$, the partial derivatives of both sides, with respect to I , implies that $u_{c_k} \left(\sum_{k=1}^K u_{G c_k} \cdot \frac{\partial \hat{c}_k}{\partial I} + u_{GG} \cdot \frac{\partial \hat{G}}{\partial I} \right) = u_G \left(\sum_{k=1}^K u_{c_k' c_k} \cdot \frac{\partial \hat{c}_k}{\partial I} + u_{c_k' G} \cdot \frac{\partial \hat{G}}{\partial I} \right)$.

That is equation ($\wedge$)

$\frac{\partial \hat{c}_{k'}}{\partial I} \cdot (u_{G c_{k'}} \cdot u_{c_{k'}} - u_{c_{k'} c_{k'}} \cdot u_G) + \sum_{k \neq k'} \left(\frac{\partial \hat{c}_k}{\partial I} (u_{c_k'} \cdot u_{G c_k} - u_G \cdot u_{c_k c_{k'}}) \right) = \frac{\partial \hat{G}}{\partial I} \cdot (u_{c_{k'} G} \cdot u_G - u_{GG} \cdot u_{c_{k'}})$ Since $\frac{\partial MRS_{G-c_{k'}}}{\partial c_k} = 0, \forall k \neq k'$, we have $\frac{\partial MRS_{G-c_{k'}}}{\partial c_k} = 0$ which means $u_{G c_k} \cdot u_{c_{k'}} - u_{c_k c_{k'}} \cdot u_G = 0$. Combining the latter with $u_{c_k' G} \cdot u_{c_{k'}} - u_{c_k' c_{k'}} \cdot u_G < 0$ and $u_{GG} \cdot u_{c_{k'}} - u_{c_k' G} \cdot u_G < 0$, we have that $\frac{\partial \hat{c}_{k'}}{\partial I} > 0$ and $\frac{\partial \hat{G}}{\partial I} < 0$. Thus, for all $k = 1, \dots, K$ $(\hat{c}_k)_{k=1}^K$ are strictly normal goods and $\hat{G}$ is an inferior good. $\square$

2. Now assume that $\frac{\partial \hat{G}}{\partial p_G} < 0$ and $\frac{\partial \hat{c}_k}{\partial p_k} > 0$ for every $k = 1, \dots, K$.

Since the utility function of each consumer is strictly quasi-concave, it follows that by the convexity of the preference relation induces by the utility $u(\hat{c}_{-k}, c_k, \hat{G}) = \hat{U}$ is strictly quasi-concave in (c_k, G) which induces a strictly convex preference relation on $(0, W] \times [0, W)$. Since, $U(c_k, G) \equiv u(\hat{c}_{-k}, c_k, G)$, as $u_k = \frac{\partial}{\partial c_k} U(c_k, G) = \frac{\partial}{\partial c_k} u(\hat{c}_{-k}, c_k, G)$ it follows that $U_k = \frac{\partial U(c_1, \dots, c_K, G)}{\partial c_k} |_{c_{-k} = \hat{c}_{-k}}$ and as we are only interested in $MRS_{G-c_k}^U |_{\wedge} = MRS_{G-c_k}^u |_{\wedge} = \frac{dc_k(G)}{dG} |_{\hat{G}}$, we will use the function $u(\hat{c}_{-k}, c_k, G)$ to finish the proof.

By (Mas-Colell et al. 1995 p. 938 Example M.D.2), we have that the determinant of the bordered Hessian satisfies:

$$\begin{vmatrix} U_{c_k c_k} & U_{c_k G} & U_{c_k} \\ U_{c_k G} & U_{GG} & U_G \\ U_{c_k} & U_G & 0 \end{vmatrix} > 0,$$

the determinant equals:

$$2U_{c_k} U_{c_k G} U_G - U_{c_k c_k} U_G U_G - U_{c_k} U_{GG} U_{c_k} > 0.$$

Note that all partial derivatives of U are actually the corresponding partial derivatives of u .

Which is $U_G(U_{c_k} U_{c_k G} - U_{c_k c_k} U_G) + U_{c_k}(U_{c_k G} U_G - U_{c_k} U_{GG}) > 0$

By the derivation of the F.O.C. with respect to I , we get equation ($\wedge$) which yields that $(U_{c_k} U_{c_k G} - U_{c_k c_k} U_G)$ and $(U_{c_k G} U_G - U_{c_k} U_{GG})$ have opposite signs, combining with $U_G < 0$ and $U_{c_k} > 0$ and the last inequality, we may conclude that $(U_{c_k G} U_G - U_{c_k} U_{GG}) > 0$ and $(U_{c_k} U_{c_k G} - U_{c_k c_k} U_G) < 0$. Thus, the strict ordinal normality conditions for the utility u holds as $u_{c_k} = U_{c_k}$, $u_{c_k G} = U_{c_k G}$ and $u_G = U_G$ where all other coordinates are $\hat{c}_{-k}$. Note that for each $(c, G) \in (0, W]^K \times [0, W)$ by the strict quasi-concavity and differentiability, we have unique supporting hyperplane through (c, G) such that $(c, G) = (\hat{c}, \hat{G})$ is the interior solution for these prices. $\square$

6 Appendix B: Uniqueness of Nash equilibrium

Under assumptions (A.1.) and (A.2.) our pollutive economy admits a unique Nash equilibrium.

Proof Assume by negation that there are two Nash equilibria s^* and s^{**} , and we further assume $G^* \geq G^{**}$. Thus, if $\forall i \in N, \forall k \in \{1, \dots, K\}, c_k^{i*} \leq c_k^{i**}$, then obviously by summation $s^* = s^{**}$. Else, $\exists i \in N, \exists k \in \{1, \dots, K\}, c_k^{i*} > c_k^{i**} > 0$. Since, $\omega_{i,k} \geq c_k^{i*} > c_k^{i**}$, it follows that for s^{**} , from the Nash maximization problem for the interior solution c_k^{i**} , we have $u_{c_k'}^i(c_i^{**}, G^{**}) + u_G^i(c_i^{**}, G^{**}) = 0$, implying $MRS_{G-c_k'}^i(c_i^{**}, G^{**}) = 1$. By (A.2), we have that $MRS_{G-c_k'}^i(c_i^*, G^*) > MRS_{G-c_k'}^i(c_i^{**}, G^{**}) = 1$. Therefore, $u_{c_k'}^i(c_i^*, G^*) + u_G^i(c_i^*, G^*) < 0$ implying there is $\varepsilon > 0$ with $\omega_k^i \geq c_{i,k}^* - \varepsilon > 0$ such that $u^i(c_i^* - \varepsilon e_k, G^* - \varepsilon) > u^i(c_i^*, G^*)$ contradicting that s^* is a Nash equilibrium. $\square$

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